

January 8, 1863.

Major-General SABINE, President, in the Chair.

The following communications were read :—

- I. “Applications of the Theory of the Polyedra to the Enumeration and Registration of Results.” By the Rev. THOMAS P. KIRKMAN, M.A., F.R.S., and Honorary Member of the Literary and Philosophical Societies of Manchester and Liverpool. Received November 29, 1862.

The following are a portion of my Tables of polyedra. They comprise all the 6-edra, 7-edra, and 8-edra, with their reciprocals, and all 9-edra of less than 17 edges. It is desirable that examples of results should be before the reader of my work on this theory, if it is so fortunate as to be read at all. More results can easily be added, if it is thought necessary, when the entire treatise is before the world.

The method of computation of these Tables turns to advantage a division of summits and reticulations not mentioned in the theory, as it would have abbreviated nothing, and would have added one more to complications already too numerous, and all inevitable.

Perfect summits and reticulations, that is, such as have no effaced effaceables, are *pyramidal*, *propyramidal*, or *metapyramidal*.

A *pyramidal* perfect summit or reticulation is one of which every effaceable, primary or secondary, is a base edge of a pyramid, by which the A-gonal base of a pyramid can be cut away in the process of reduction of the reticulation. Such a construction is made either by glueing together by their edges A-gonal, B-gonal, C-gonal, &c. pyramidal bases, the vertices being supposed to hang downwards, or by loading marginal triangles of plane reticulations with such bases so posited.

If a reticulation has no effaceables, it is merely a plane partitioned polygon, and the summit or edge which crowns it completes a polyedron without the employment of any solid charges. I call such a summit or edge *propyramidal*.

If, for one or more of the A-gonal, B-gonal, &c. bases of pyramids

about a pyramidal summit or edge, we substitute an A-gonal, B-gonal, &c. face of any polyedron which is not a pyramidal base, we have a perfect summit or edge of a solid of a greater number of edges, which may or may not have another symmetry. Such a summit or edge is *metapyramidal*.

The most expeditious method of computing the polyedra of N or fewer edges, is first to form and to crown all possible propyramidal and pyramidal perfect reticulations which can be reduced by effacement of effaceables to N or to fewer edges: these are to be registered in tables of perfect edges and summits, which show at a glance what pyramidal bases enter into the constructions registered. Having determined, by inspection of these tables and by effacements, the lower polyedra, we form tables of metapyramidal edges and summits, by merely conceiving the substitution of other A-gons, B-gons, &c. of solids thus far determined, for the A-gonal, B-gonal, &c. pyramidal bases. The edge (MN), or the *p*-ace, considered, is at once entered as an edge (MN), or as a *p*-ace, of a polyedron of more edges, in the metapyramidal tables.

Rules are easily laid down for the result of this conceived substitution, as to symmetry, signatures, and the tabular value (of enumeration). We thus escape the enormous toil of separately constructing and crowning the metapyramidal reticulations. These rules will be given in the *supplement of applications*, of which this abstract exhibits a few results.

I observe that a case is unprovided for in art. XLVII. of my second section, namely, the case of a zone which exhibits in some of its forms the symbol  $0_p$  of an epizonal polar edge. Such a zone will of course occur about an amphigrammic, about an edrographic, or gono-graphic zoned axis. This polar epizonal is to be included, as part of the number  $h_{AA}$ , in the sinister of the equation first read, since this edge  $0_p$  is generally the epizonal edge of a monozone A-gon, never of two different A-gons.

This XLVIIth article is sufficiently corrected by the effacement of the word *non-polar* in the 2nd, 9th, and 13th lines, and by writing *when* for *because* in the 7th line. So read, all cases are covered.

It may appear to the reader at first sight that the Table A, or at most the Tables A, B, and C, would comprise a sufficient solution of the problem of the polyedra. The truth is, that it is impossible to

determine the numbers in the Table A, without complete Tables A, B, C, D of inferior polyedra.

*Registration of the 4-edron 4-acron.*

(*Vide* arts. XXXI. &c. of my memoir "On the Theory of the Polyedra," Phil. Trans. 1862.)

Table A.

One zoned tetrarchaxine, having principal polar triaces and triangles, and amphigrammic secondary axes (art. XXI.). The zone is  ${}^4Z = \{2.1_p, 2.1_p, 0_p 0_p\}$ .

Table B.

*Janal polar zoned edge :*

$$(33)_{ja.hom}^{2a.d} 02 = 1, {}^4Z = \{2.1_p, 2.1_p, 0_{p_1}, 0_{p_1}\}.$$

Table C.

*Zoned radical tetrarchipolar face :*

$$({}^43)_{go.ed}^{3mo} 13 = 1, {}^4Z = \{2.1_p, 2.1_p, 0_p 0_p\}.$$

The 4 prefixed to 3 shows that it is a tetrarchaxine pole: the 3 suffixed shows that the summits of the polar triangle are triaces. It is only in the case of polar triangles that we require, for purposes of construction hereafter, an account of summits about a polar face.

Table D.

*Polar zoned edge :*

$$(33)_{am.gr}^{2a.d} 02 = 1, Z = Z' = \{2, 2, 0_p, 0_p\}.$$

This is the edge above recorded in Table B.

The summit reciprocal to the face in Table C is the one required to complete that Table. The reciprocal of any face is written by exchanging faces for summits, and zonal for epizonal edges, in all the signatures. Hence the summit of the 4-edron is

$$3_{go.ed}^{3mo} 13 = 1, Z = \{2.1_p, 2.1_p, 0_p 0_p\}.$$

*Registration of the 5-edron 5-acron.*

Table A.

One 4-zoned monaxine heteroid, the 4-gonal pyramid.

Table C.

*Zoned polar face :*

$$4_{go.ed}^{4a.d} 14 = 1, Z = \{1_p + 2.1, 1_p, 0^{2.1}\}, \\ Z' = \{1_p, 1_p + 2.1, 0^{2.1}\}.$$

*Zoned polar summit:*

$$4_{goed}^{4a.d}14=1, \text{ with the same zones.}$$

*Zoned non-polar or monozone face:*

$$3^{mo}24=1, \quad Z=\{1, 3, 0^2\}.$$

*Zoned summit:*

$$3^{mo}24=1, \quad Z=\{3, 1, 0^2\}.$$

Table D.

*Zonal edge:*

$$(33)_{zo}13=1, \quad Z=\{3, 1, 0^2\}.$$

*Epizonal edge:*

$$(43)_{ep}03=1, \quad Z=\{1, 3, 0^2\}.$$

*Registration of the 5-edron 6-acron.*

Table A.

One 3-zoned monarchaxine, having an amphiedral principal axis terminated by triangles, and edrographic secondary axes. Its two zones are those next written.

Table B.

*Janal zoned polar face:*

$$(3)3_{amed}^{3mo}34=1, \quad Z=\{2.1, 2.1_p+1_p, 0_p, 0^{2.1}\}, \\ Z'=3\{\dots 1_p, 0_p\}.$$

Table C.

*Polar zoned faces:*

$$(3)3_{amed}^{mo}34=1, \quad Z=\{2, \quad 1_p+2, \quad 0, \quad 0^2\}; \\ 4_{edgr}^{2ag}24=1, \quad Z=\{2.1, \quad 1_p+2.1, \quad 0_p, \quad 0^{2.1}\}, \\ Z'=\{\dots 1_p+2.1, \quad 0_p, \quad 0^{2.1}\}.$$

Table D.

*Polar zoned edge:*

$$(44)_{edgr}^{2a.d}03=1, \quad Z=\{2.1, \quad 1_p+2.1, \quad 0_p, \quad 0^{2.1}\}, \\ Z'=\{\dots 1_p+2.1, \quad 0_p, \quad 0^{2.1}\}.$$

*Epizonal edge:*

$$(34)_{ep}13=1, \quad Z=\{2, \quad 3, \quad 0, \quad 0^2\}.$$

We have not here registered the summits of the 5-edron 6-acron, as they are merely the reciprocals of the faces of the 6-edron 5-acron. For a like reason we shall avoid the trouble of registering hereafter any summits.

*Registration of the 6-edron 5-acron.*

## Table A.

One 3-zoned monarchaxine, having an amphigonal principal axis, terminated by triaces, and gonogrammic secondary axes, with the zones first written below in Table D.

## Table C.

*Zoned non-polar face:*

$$3^{mo}25=1, \quad Z=\{3, 2, 0, 0^2\}.$$

## Table D.

*Polar zoned edge:*

$$(33)_{gogr}^{2a.d}14=1, \quad Z=\{1_p+2.1, 2.1, 0_p, 0^{2.1}\}, \\ Z'=\{1_p+2.1, \dots, 0_p, 0^{2.1}\}.$$

*Zonal edge:*

$$(33)_{zo}14=1, \quad Z=\{3, 2, 0, 0^2\}.$$

*Registration of the 6-edra 6-acra.*

## Table A.

1. One 5-zoned monaxine heteroid, whose gonoedral axis is terminated by a pentace and a pentagon, having the zone first below written.

2. One 2-ple zoneless monaxine heteroid, having an amphigrammic axis.

## Table C.

*Polar face:*

$$5_{gono}^{5mo}15=1, \quad Z=\{1_p+1, 1_p+1, 0, 0\}.$$

*Zoned non-polar face:*

$$3^{mo}35=1, \quad Z=\{2, 2, 0, 0\}.$$

*Asymmetric faces:*

$$3_{as}35=2, \quad 4_{as}25=1.$$

## Table D.

*Zoneless polar edges:*

$$(44)_{angr}^204=1, \quad (33)_{angr}^224=1.$$

*Zonal edge:*

$$(33)_{zo}24=1, \quad Z=\{2, 2, 0, 0\}.$$

*Epizonal edge:*

$$(35)_{ep}04=1, \quad Z=\{2, 2, 0, 0\}.$$

*Asymmetric edges:*

$$(34)_{as}14=3, \quad (33)_{as}24=1.$$

*Registration of the 7-edra 6-acra.*

Table A.

1. One 2-zoned monaxine heteroid, having an edrographic axis, and the zones first below written in Table C.

2. One monozone, having the zone  $Z = \{2, 3, 0, 0^2\}$ .

Table C.

*Polar zoned face:*

$$4_{edgr}^{2di} 26 = 1, \quad Z = \{2.1, 1_p + 2.1, 0_p\}, \\ Z' = \{2.2, 1_p, 0_p, 0^{2.1}\}.$$

*Zoned non-polar faces:*

$$4^{ag} 26 = 1, \quad Z = \{2, 3, 0, 0^2\}; \\ 3^{mo} 36 = 2, \quad Z = \{2, 3, 0, 0^2\}; \\ 3^{mo} 36 = 1, \quad Z = \{2, 3, 0\}.$$

*Asymmetric faces:*

$$3_{as} 36 = 3.$$

Table D.

*Polar zoned edge:*

$$(33)_{edgr}^{2a.d} 25 = 1, \quad Z = \{2.1, 1_p + 2.1, 0_p\}, \\ Z' = \{2.2, 1_p, 0_p, 0^{2.1}\}.$$

*Zonal edges:*

$$(33)_{z0} 25 = 1, \quad Z = \{4, 1, 0^3\}; \\ (33)_{z0} 25 = 1, \quad Z = \{2, 3, 0, 0^2\}.$$

*Epizonal edges:*

$$(34)_{ep} 15 = 2, \quad Z = \{2, 3, 0, 0^2\}.$$

*Asymmetric edges:*

$$(33)_{as} 25 = 4, \quad (34)_{as} 15 = 2.$$

*Registration of the 6-edra 7-acra.*

Table A.

1. One 2-zoned monaxine heteroid, with gonographic axis, having the zones first read in Table D below.

2. One monozone, with the zone  $Z = \{3, 2, 0, 0^2\}$ .

Table C.

*Zoned non-polar faces:*

$$5^{mo} 25 = 1, \quad Z = \{3, 2, 0, 0^2\}; \\ 4^{ag} 35 = 1, \quad Z = \{1, 4, 0^3\}; \\ 4^{di} 35 = 1, \quad Z = \{3, 2, 0\}; \\ 3^{mo} 45 = 1, \quad Z = \{1, 4, 0^3\}; \\ 3^{mo} 45 = 1, \quad Z = \{3, 2, 0^2, 0\}.$$

*Asymmetric faces :*

$$4_{as}35=1, \quad 3_{as}45=1.$$

Table D.

*Zoned polar edge :*

$$(44)_{gogr}^{2a,d}14=1, \quad Z=\{1_p+2.1, \quad 2.1, \quad 0_p\}, \\ Z'=\{1_p, \quad 2.2, \quad 0_p, \quad 0^{2.1}\}.$$

*Zonal edges :*

$$\left. \begin{array}{l} (33)_{zo}34=1 \\ (44)_{zo}14=1 \end{array} \right\} \quad Z=\{3, \quad 2, \quad 0, \quad 0^2\}.$$

*Epizonal edges :*

$$(35)_{ep}14=1, \quad Z=\{3, \quad 2, \quad 0, \quad 0^2\}; \\ (34)_{ep}24=1, \quad Z=\{1, \quad 4, \quad 0^3\}.$$

*Asymmetric edges :*

$$(53)_{as}14=1, \quad (34)_{as}24=3; \\ (45)_{as}04=1, \quad (44)_{as}14=1.$$

*Registration of 7-edra 7-acra.*

Table A.

1. One 6-zoned monaxine heteroid, having its gonoeedral axis terminated by a hexace and a hexagon, and the zones first below written in Table C.

2. Two 3-zoned monaxine heteroids, with gonoeedral axes, having one the zone  $Z=\{1_p+2, \quad 1_p+2, \quad 0, \quad 0\}$ , and the other the zone  $Z=\{1_p+2, \quad 1_p+2, \quad 0^2, \quad 0^2\}$ . Each axis is terminated by a triace and a triangle.

3. One 2-ple zoneless monaxine heteroid, with gonoeedral axis, terminated by a 4-ace and a 4-gon.

4. Two monozones, having one the zone  $Z=\{1, \quad 3, \quad 0^2\}$ , and the other  $Z=\{3, \quad 1, \quad 0^2\}$ .

5. Two asymmetric 7-edra 7-acra.

Table C.

*Polar zoned faces :*

$$6_{goed}^{6a,d}16=1, \quad Z=\{1_p, \quad 1_p+2.1, \quad 0^2\}, \\ Z'=\{1_p+2.1, \quad 1_p, \quad 0^2\}; \\ (3)3_{goed}^{3mo}46=1, \quad Z=\{1_p+2.1, \quad 1_p+2.1, \quad 0^2, \quad 0^2\}; \\ (4)3_{goed}^{3mo}46=1, \quad Z=\{1_p+2.1, \quad 1_p+2.1, \quad 0, \quad 0\}.$$

*Zoneless polar face :*

$$4_{\text{polar}}^z 36=1.$$

*Zoned non-polar faces :*

$$5^{mo} 26=1, \quad Z=\{1, 3, 0^2\};$$

$$4^{ag} 36=1, \quad 3^{mo} 47=2, \quad Z=\{1, 3, 0^2\};$$

$$4^{ag} 36=1, \quad 3^{mo} 47=1, \quad Z=\{3, 3, 0^2, 0^2\};$$

$$4^{di} 36=1, \quad 3^{mo} 47=1, \quad Z=\{3, 3, 0, 0\};$$

$$4^{di} 36=1, \quad Z=\{3, 1, 0^2\}.$$

*Asymmetric faces :*

$$5_{as} 26=1, \quad 4_{as} 36=6, \quad 3_{as} 46=15.$$

Table D.

*Zonal edges :*

$$(44)_{zo} 15=1, \quad Z=\{3, 3, 0^2, 0^2\};$$

$$(44)_{zo} 15=1, \quad Z=\{3, 3, 0, 0\};$$

$$(44)_{zo} 15=1, \quad Z=\{3, 1, 0^2\};$$

$$(33)_{zo} 15=1, \quad Z=\{3, 3, 0^2, 0^2\};$$

$$(33)_{zo} 15=2, \quad Z=\{3, 1, 0^2\}.$$

*Epizonal edges :*

$$(45)_{ep} 05=1, \quad Z=\{1, 3, 0^2\};$$

$$(36)_{ep} 05=1, \quad Z=\{1, 3, 0^2\};$$

$$(34)_{ep} 25=2, \quad Z=\{3, 3, 0^2, 0^2\};$$

$$(34)_{ep} 25=1, \quad Z=\{1, 3, 0^2\};$$

$$(33)_{ep} 35=1, \quad Z=\{3, 3, 0, 0\}.$$

*Asymmetric edges :*

$$(44)_{as} 15=4, \quad (45)_{as} 05=1,$$

$$(33)_{as} 35=10, \quad (34)_{as} 25=20, \quad (35)_{as} 15=6.$$

*Registration of 6-edra 8-acra.*

Table A.

1. One zoned triarchaxine, having the zones first below read, with three 4-zoned amphiedral janal principal, four objanal amphigonal 3-zoned secondary, and six amphigrammic janal tertiary axes.

2. One 2-zoned monaxine heteroid, with amphigrammic axis, whose zones are read below in Table D.

Table B.

*Radical zoned triarchipole:*

$$4_{ja.rad}^{4ad}45=1, \begin{cases} {}^3Z=\{4.1_{p_2}, 2.1_{p_1}, 0_{p_3}^{2.1}\}, \\ Z'=\{.., 4.1_{p_1}, 0_{p_3}^{4.1}\}. \end{cases}$$

*Janal zoned polar edge:*

$$(44)_{ja}^{2a.d}24=1, \quad \begin{cases} Z=\{2.2, 2.1, 0_p^2\}, \\ Z'=\{.., 2.2, 0_p^2, 0^2\}. \end{cases}$$

Table C.

*Zoned non-polar faces:*

$$\begin{aligned} 5^{mo}35=1, \quad Z &= \{2, 2, 0, 0\}; \\ 4^{ag}45=1, \quad Z &= \{2, 4, 0, 0^3\}; \\ 3^{mo}55=1, \quad Z &= \{2, 4, 0, 0^3\}. \end{aligned}$$

Table D.

*Zoned polar edge:*

$$\begin{aligned} (55)_{am.gr}^{2a.d}04=1, \quad & \left\{ \begin{aligned} Z &= \{2.1, 2.1, 0_p^*, 0_p\}; \\ (44)_{am.gr}^{2a.d}24=1, \quad & Z' = \{2.1, 2.2, 0_p, 0_p, 0^{2.1}\}. \end{aligned} \right. \\ (44)_{am.gr}^{2a.d}24=1, \quad & \begin{cases} Z = \{2.2, 2.1, 0_p^2\}, \\ Z' = \{.., 2.2, 0_p^2, 0^{2.1}\}. \end{cases} \end{aligned}$$

*Epizonal edge:*

$$(34)_{ep}34=1, \quad Z = \{2, 4, 0, 0^3\}.$$

*Asymmetric edges:*

$$(54)_{as}14=1, \quad (53)_{as}24=1.$$

*Registration of 8-edra 6-acra.*

Table A.

1. One zoned triarchaxine, whose three principal 4-zoned amphigonal axes carry 4-aces, and have each the zones

$${}^3Z = \{2.1_{p_1}, 4.1_p, 0_{p_3}^{2.1}\}, \quad {}^3Z' = \{4.1_{p_1}, .., 0_{p_3}^{4.1}\},$$

whose four objanal amphiedral 3-zoned secondary axes have the zone first below read, and which has six tertiary janal amphigrammic axes, carrying the above-written zones.

2. One 2-zoned monaxine heteroid, with zones read below in Table D, about an amphigrammic axis.

Table B.

*Homozone polar face :*

$$\begin{aligned} 3_{obja}^{mo}37=1, \quad {}^3Z=\{2.1, 2(1_p+1), 0^{2.1}\}, \\ \zeta=6\{0_p\} \quad (\text{art. XXII.}). \end{aligned}$$

This 3-zoned secondary pole of the regular octaedron is registered as the termination of a homozone axis ; for all janal constructions upon it will be homozones. The six poles registered in the zonoid signature are here *zoned* amphigrammic poles of 2-ple repetition. To see this, we need only crown the hexagon 123456 with a triace above on 135, and with a triace below on 246 : the axis thus constructed is the reciprocal of the one here recorded.

*Janal zoned polar edge :*

$$(33)_{ja}^{2a.d}26=1, \quad \begin{cases} Z=\{2.1, 2.2, 0_p^{2.1}\}, \\ Z'=\{2.2, \dots, 0_p^{2.1}, 0^{2.1}\}. \end{cases}$$

Table C.

*Zoned non-polar faces :*

$$\begin{aligned} 3^{mo}37=1, \quad Z=\{2, 2, 0, 0\}; \\ 3^{mo}37=1, \quad Z=\{4, 2, 0^3, 0\}. \end{aligned}$$

*Asymmetric face :*

$$3_{as}37=1.$$

Table D.

*Zoned polar edges :*

$$(33)_{am.gr}^{2a.d}26=2, \quad \begin{aligned} Z &= \{2.1, 2.1, 0_p, 0_p\}, \\ Z' &= \{2.2, 2.1, 0_p, 0_p, 0^{2.1}\}. \end{aligned}$$

*Zonal edge :*

$$(33)_{zo}26=1, \quad Z=\{4, 2, 0^3, 0\}.$$

*Asymmetric edges :*

$$(33)_{as}26=2.$$

*Registration of 7-edra 8-acra.*

Table A.

1. Two 2-zoned monaxine heteroids, having each an edrographic axis, one terminated by the polar hexagon, and the other by the polar tetragon, first below written.

2. Two 2-ple zoneless monaxine heteroids, with edrographic axes, both terminated by tetragons.

3. Four monozones : three of them have the zone  $Z = \{2, \mathbf{3}, 0^2, 0\}$  ; the fourth has the zone  $Z = \{2, \mathbf{3}, 0\}$ .

4. Three asymmetric 7-edra 8-acra.

Table C.

*Zoned polar faces :*

$$\begin{aligned} 6_{edgr}^{2a.d} 26 &= 1, & Z &= \{ \dots, 1_p + 2.1, 0_p, 0^{2.1} \}, \\ & & Z' &= \{ 2.2, 1_p, 0_p, 0^{2.1} \}. \\ 4_{edgr}^{2di} 46 &= 1, & Z &= \{ 2.2, 1_p + 2.1, 0_p, 0^{2.1} \}, \\ & & Z' &= \{ 2.2, 1_p, 0_p, 0^{2.1} \}. \end{aligned}$$

*Zoneless polar faces :*

$$4_{edgr}^2 46 = 2.$$

*Zoned non-polar faces :*

$$\begin{aligned} 6^{ag} 26 &= 1, & Z &= \{ 2, \mathbf{3}, 0, 0^2 \} ; \\ 5^{mo} 36 &= 2, & Z &= \{ 2, \mathbf{3}, 0, 0^2 \} ; \\ 5^{mo} 36 &= 1, & Z &= \{ 2, \mathbf{3}, 0 \} ; \\ 4^{ag} 46 &= 1, & Z &= \{ \dots, \mathbf{3}, 0^3 \} ; \\ 4^{ag} 46 &= 2, & Z &= \{ 2, \mathbf{3}, 0, 0^2 \} ; \\ 4^{di} 46 &= 1, & Z &= \{ 2, \mathbf{3}, 0 \} ; \\ 3^{mo} 56 &= 4, & Z &= \{ 2, \mathbf{3}, 0, 0^2 \} ; \\ 3^{mo} 56 &= 1, & Z &= \{ 2, \mathbf{3}, 0 \} ; \\ 3^{mo} 56 &= 1, & Z &= \{ 4, \mathbf{3}, 0^2, 0 \}. \end{aligned}$$

*Asymmetric faces :*

$$5_{us} 36 = 5, \quad 4_{us} 46 = 14, \quad 3_{us} 56 = 18.$$

Table D.

*Zoned polar edges :*

$$\begin{aligned} (44)_{edgr}^{2a.d} 25 &= 1, & Z &= \{ 2.2, 1_p, 0_p, 0^{2.1} \}, \\ & & Z' &= \{ \dots, 1_p + 2.1, 0, 0^{2.1} \} ; \\ (33)_{edgr}^{2a.d} 45 &= 1, & Z &= \{ 2.2, 1_p, 0_p, 0^{2.1} \}, \\ & & Z' &= \{ 2.2, 1_p + 2.1, 0_p, 0^{2.1} \}. \end{aligned}$$

*Zoneless polar edges :*

$$(55)_{edgr}^2 05 = 1, \quad (44)_{edgr}^2 25 = 1.$$

*Zonal edges :*

$$\begin{aligned} (44)_{zo} 25 &= 2, & (33)_{zo} 45 &= 1, & Z &= \{ 2, \mathbf{3}, 0, 0^2 \} ; \\ & & (44)_{zo} 25 &= 1, & Z &= \{ 4, \mathbf{3}, 0^2, 0 \} ; \\ (44)_{zo} 25 &= 1, & (33)_{zo} 45 &= 1, & Z &= \{ 4, \mathbf{1}, 0^3 \}. \end{aligned}$$

*Epizonal edges :*

$$\begin{aligned}
 (46)_{ep}05 &= 1, & Z &= \{ \dots, 3, 0^3 \}; \\
 (45)_{ep}15 &= 2, & (36)_{ep}15 &= 2, & (34)_{ep}35 &= 2, & Z &= \{ 2, 3, 0, 0^2 \}; \\
 (35)_{ep}25 &= 1, & Z &= \{ 2, 3, 0 \}.
 \end{aligned}$$

*Asymmetric edges :*

$$\begin{aligned}
 (64)_{as}05 &= 1, & (55)_{as}05 &= 1, & (45)_{as}15 &= 11, & (36)_{as}15 &= 2; \\
 (44)_{as}25 &= 11, & (35)_{as}25 &= 17, & (34)_{as}35 &= 25, & (33)_{as}45 &= 8.
 \end{aligned}$$

*Registration of 8-edra 7-acra.*

## Table A.

1. Two 2-zoned monaxine heteroids, with gonogrammic axes, terminated by a hexace having the zones first below read in Table D, and by a tesseract with the zones next there written.

2. Two 2-ple zoneless monaxine heteroids with gonogrammic axes, both terminated by tesseraces.

3. Four monozones; three having the zone  $Z = \{3, 2, 0^2, 0\}$ , and one with the zone  $Z = \{3, 2, 0\}$ .

4. Three asymmetric 8-edra 7-acra.

## Table C.

*Zoned non-polar faces :*

$$\begin{aligned}
 5^{mo}27 &= 2, & Z &= \{3, 2, 0^2, 0\}; \\
 4^{di}37 &= 2, & Z &= \{3, 2, 0\}; \\
 4^{ag}37 &= 2, & Z &= \{1, 4, 0^3\}; \\
 3^{mo}47 &= 4, & Z &= \{3, 2, 0^2, 0\}; \\
 3^{mo}47 &= 2, & Z &= \{1, 4, 0^3\}; \\
 3^{mo}47 &= 1, & Z &= \{3, 4, 0, 0^2\}.
 \end{aligned}$$

*Asymmetric faces :*

$$4_{as}37 = 9, \quad 3_{as}47 = 36.$$

## Table D.

*Zoned polar edges :*

$$\begin{aligned}
 (44)_{go.gr}^{2a.d}16 &= 1, & Z &= \{1_p + 2.1, 2.2, 0_p, 0^{2.1}\}, \\
 & & Z' &= \{1_p, 2.2, 0_p, 0^{2.1}\}; \\
 (44)_{go.gr}^{2a.d}16 &= 1, & Z &= \{1 + 2.1, \dots, 0_p, 0^{2.1}\}, \\
 & & Z' &= \{1_p, 2.2, 0_p, 0^{2.1}\}.
 \end{aligned}$$

*Zoneless polar edges :*

$$(33)_{go.gr}^246 = 2.$$

*Zonal edges :*

$$\begin{aligned}
 (44)_{zo}16=1, & \quad (33)_{zo}36=5, & Z=\{3, 2, 0^2, 0\}; \\
 (33)_{zo}36=1, & & Z=\{3, 2, 0\}; \\
 (33)_{zo}36=1, & & Z=\{3, \dots, 0^3\}.
 \end{aligned}$$

*Epizonal edges :*

$$\begin{aligned}
 (35)_{ep}16=2, & \quad (33)_{ep}36=1, & Z=\{3, 2, 0^2, 0\}; \\
 (34)_{ep}26=2, & & Z=\{1, 4, 0^3\}; \\
 (33)_{ep}36=1, & & Z=\{3, 4, 0, 0^2\}.
 \end{aligned}$$

*Asymmetric edges :*

$$\begin{aligned}
 (35)_{as}16=4, & \quad (33)_{as}36=34; \\
 (44)_{as}16=3, & \quad (34)_{as}26=35.
 \end{aligned}$$

*Registration of 7-edra 9-acra.*

## Table A.

1. Two 2-zoned monaxine heteroids, with gonoedral axes; one carrying a hexagon and a tessarace, and the zones first below read; the other carrying a tetragon and a tessarace, with the zones next written.

2. Four monozones; one with the zone  $Z=\{1, 3, 0^2\}$ , one with  $Z=\{3, 3, 0^2, 0^2\}$ , and two with  $Z=\{3, 3, 0, 0\}$ .

3. Two asymmetric 7-edra 9-acra.

## Table C.

*Zoned polar faces :*

$$\begin{aligned}
 6_{goed}^{2a,d}36=1, & \quad Z=\{1_p+2.1, 1_p+2.1, 0^{2.1}, 0^{2.1}\}, \\
 & \quad Z'=\{1_p+2.1, 1_p, 0^{2.1}\}; \\
 4_{goed}^{2ag}56=1, & \quad Z=\{1_p, 1_p+2.2, 0^{2.2}\}, \\
 & \quad Z'=\{1_p, 1_p+2.1, 0^{2.1}\}.
 \end{aligned}$$

*Zoned non-polar faces :*

$$\begin{aligned}
 6^{ag}36=1, & \quad Z=\{1, 3, 0^2\}; \\
 5^{mo}46=2, & \quad Z=\{1, 3, 0^2\}; \\
 5^{mo}46=2, & \quad Z=\{3, 3, 0, 0\}; \\
 5^{mo}46=1, & \quad Z=\{3, 3, 0^2, 0^2\}; \\
 4^{di}56=2, & \quad Z=\{3, 3, 0, 0\}; \\
 4^{ag}56=1, & \quad Z=\{3, 3, 0^2, 0^2\}; \\
 4^{ag}56=1, & \quad Z=\{1, 5, 0^4\}; \\
 3^{mo}66=2, & \quad Z=\{3, 3, 0, 0\}; \\
 3^{mo}66=2, & \quad Z=\{3, 3, 0^2, 0^2\}; \\
 3^{mo}66=1, & \quad Z=\{1, 3, 0^2\}; \\
 3^{mo}66=1, & \quad Z=\{1, 5, 0^4\}.
 \end{aligned}$$

*Asymmetric faces :*

$$6_{as}36=1, \quad 5_{as}46=5, \quad 4_{as}56=9, \quad 3_{as}66=8.$$

Table D.

*Zonal edges :*

$$\begin{aligned} (55)_{zo}15=1, \quad (44)_{zo}35=1, \quad Z=\{3, 3, 0, 0\}; \\ (33)_{zo}55=1, \quad (55)_{zo}15=1, \quad (44)_{zo}35=1, \quad Z=\{3, 3, 0^2, 0^2\}; \\ (44)_{zo}35=1, \quad Z=\{3, 1, 0^2\}. \end{aligned}$$

*Epizonal edges :*

$$\begin{aligned} (54)_{ep}25=1, \quad (36)_{ep}25=1, \quad (34)_{ep}45=1, \quad Z=\{3, 3, 0^2, 0^2\}; \\ (56)_{ep}05=1, \quad (36)_{ep}25=1, \quad (45)_{ep}25=1, \quad Z=\{1, 3, 0^2\}; \\ (35)_{ep}35=2, \quad Z=\{3, 3, 0, 0\}; \\ (44)_{ep}35=1, \quad (34)_{ep}45=1, \quad Z=\{1, 5, 0^4\}. \end{aligned}$$

*Asymmetric edges :*

$$\begin{aligned} (65)_{as}05=1, \quad (55)_{as}15=3, \quad (64)_{as}15=4, \\ (63)_{as}25=4, \quad (54)_{as}25=13, \quad (53)_{as}35=13, \\ (44)_{as}35=12, \quad (43)_{as}45=10, \quad (33)_{as}55=1. \end{aligned}$$

*Registration of 9-edra 7-acra.*

Table A.

1. Two 2-zoned monaxine heteroids, having gonoedral axes, one carrying a hexace and a tetragon, with the zones first below read, and the other carrying a tetragon and a tesseract, with the next-written zones.

2. Four monozones ; one having the zone  $Z=\{3, 1, 0^2\}$ , one having  $Z=\{3, 3, 0^2, 0^2\}$ , and two having  $Z=\{3, 3, 0, 0\}$ .

3. Two asymmetric 9-edra 7-acra.

Table C.

*Zoned polar faces :*

$$\begin{aligned} 4_{go.ed}^{2ag}38=1, \quad Z=\{1_p+2.1, 1_p+2.1, 0^{2.1}, 0^{2.1}\}, \\ Z'=\{1_p, 1_p+2.1, 0^{2.1}\}; \\ 4_{go.ed}^{2di}38=1, \quad Z=\{1_p+2.2, 1_p, 0^{2.2}\}, \\ Z'=\{1_p+2.1, 1_p, 0^{2.1}\}. \end{aligned}$$

*Zoned non-polar faces :*

$$\begin{aligned} 4^{di}38=1, \quad Z=\{3, 1, 0^2\}; \\ 4^{di}38=2, \quad Z=\{3, 3, 0, 0\}; \\ 4^{ag}38=1, \quad Z=\{3, 3, 0^2, 0^2\}; \\ 3^{mo}48=3, \quad Z=\{3, 3, 0^2, 0^2\}; \\ 3^{mo}48=4, \quad Z=\{3, 3, 0, 0\}; \\ 3^{mo}48=1, \quad Z=\{1, 3, 0^2\}. \end{aligned}$$

*Asymmetric faces :*

$$4_{as}36=2, \quad 3_{as}48=32.$$

Table D.

*Zonal edges :*

$$(33)_{zo}37=3, \quad Z=\{3, 3, 0^2, 0^2\};$$

$$(33)_{zo}37=2, \quad Z=\{3, 3, 0, 0\};$$

$$(33)_{zo}37=3, \quad Z=\{3, 1, 0^2\};$$

$$(33)_{zo}37=2, \quad Z=\{5, 1, 0^4\}.$$

*Epizonal edges :*

$$(33)_{ep}37=2, \quad Z=\{3, 3, 0, 0\};$$

$$(34)_{ep}27=3, \quad Z=\{3, 4, 0^2, 0^2\};$$

$$(34)_{ep}27=1, \quad Z=\{1, 3, 0^2\}.$$

*Asymmetric edges :*

$$(34)_{as}27=16, \quad (33)_{as}37=39.$$

*Registration of 8-edra 8-acra.*

Table A.

1. One zoned triaxine, having three amphigrammic axes, whose poles and zones are read below in Table B.

2. Two homozone triaxines, having one a zoned amphigrammic and two zoneless amphiedral, and the other a zoned amphigrammic and two zoneless amphigonal, axes. The poles and zones are written below in Table B.

3. One 7-zoned monaxine heteroid, viz. the pyramid on 7-gonal base.

4. Five 2-ple zoneless monaxine heteroids, with amphigrammic axes.

5. Eleven monozones, of which

$$3 \text{ have the zone } Z=\{2, 2, 0, 0\},$$

$$2 \quad \quad \quad Z=\{4, 2, 0^3, 0\},$$

$$2 \quad \quad \quad Z=\{2, 4, 0, 0^3\},$$

$$2 \quad \quad \quad Z=\{2, 4, 0^2\},$$

$$2 \quad \quad \quad Z=\{4, 2, 0^2\}.$$

6. Twenty-two asymmetric 8-edra 8-acra.

Table B.

*Janal zoneless polar face :*  $4_{ja}^247=1,$

the zoneless pole of one homozone triaxine. The reciprocal summit is the zoneless pole of the other homozone triaxine.

*Heterozone janal polar edges :*

$$(44)_{ja}^{2a.d} 26=1, \quad \{Z_1, Z_2, Z_3\},$$

$$(44)_{ja}^{2a.d} 26=1, \quad \{Z_2, Z_3, Z_1\},$$

$$(33)_{ja}^{2a.d} 46=1, \quad \{Z_3, Z_1, Z_2\},$$

where

$$Z_1=\{2.2, \dots, 0_p^2, 0_p^2\},$$

$$Z_2=\{\dots, 2.2, 0_p^2, 0_p^2\},$$

$$Z_3=\{2.2, 2.2, 0_p^2, 0_p^2\},$$

and the two zones first named after an edge are those about its axis.

*Homozone janal polar edges :*

$$(44)_{ja}^{2a.d} 26=1, \quad Z=\{2.1, 2.2, 0_p, 0_p, 0^{2.1}\},$$

$$\zeta=4\{1_p\};$$

$$(33)_{ja}^{2a.d} 46=1, \quad Z=\{2.2, 2.1, 0_p, 0_p, 0^{2.1}\},$$

$$\zeta=4\{1_p\}.$$

Table C.

*Zoned polar face :*

$$7^{7mo} 17=1, \quad Z=\{1_p+1, 1_p+1, 0, 0\}.$$

*Zoneless polar face :*

$$4_{amgo}^2 47=1.$$

*Zoned non-polar faces :*

$$5^{mo} 37=4, \quad Z=\{2, 2, 0, 0\};$$

$$5^{mo} 37=2, \quad Z=\{4, 2, 0^3, 0\};$$

$$5^{mo} 37=1, \quad Z=\{2, 4, 0^2\};$$

$$4^{ag} 47=5, \quad Z=\{2, 4, 0^3, 0\};$$

$$4^{ag} 47=1, \quad Z=\{2, 4, 0^2\};$$

$$4^{ag} 47=1, \quad Z=\{\dots, 4, 0^4\};$$

$$4^{di} 47=4, \quad Z=\{4, 2, 0^2\};$$

$$4^{di} 47=1, \quad Z=\{2, 4, 0^2\};$$

$$3^{mo} 57=3, \quad Z=\{2, 2, 0, 0\};$$

$$3^{mo} 57=3, \quad Z=\{4, 2, 0^3, 0\};$$

$$3^{mo} 57=5, \quad Z=\{2, 4, 0^2\};$$

$$3^{mo} 57=5, \quad Z=\{2, 4, 0, 0^3\};$$

$$3^{mo} 57=1, \quad Z=\{4, 4, 0^2, 0^2\}.$$

*Asymmetric faces :*

$$6_{as}27=2, \quad 5_{as}37=16, \quad 4_{as}47=70, \quad 3_{as}57=137.$$

The reciprocals of all these faces are the summits of Table C ; but we omit them, as we never have occasion, in our processes of construction, to inspect those summits.

Table D.

*Zoned polar edges :*

$$(44)_{am.gr}^{2a.d}26=1, \quad Z=\{2.2, \dots, 0_p^2, 0^{2.1}\}, \quad Z'=\{\dots, 2.2, 0_p^2, 0^{2.1}\};$$

$$(44)_{am.gr}^{2a.d}26=1, \quad Z=\{\dots, 2.2, 0_p^2, 0^{2.1}\}, \quad Z'=\{2.2, 2.2, 0_p^2, 0^{2.1}\};$$

$$(33)_{am.gr}^{2a.d}46=1, \quad Z=\{2.2, 2.2, 0_p^2, 0^{2.1}\}, \quad Z'=\{2.2, \dots, 0_p^2, 0^{2.1}\};$$

$$(44)_{am.gr}^{2a.d}26=1, \quad Z=Z'=\{2.1, 2.2, 0_p, 0_p, 0^{2.1}\};$$

$$(33)_{am.gr}^{2a.d}46=1, \quad Z=Z'=\{2.2, 2.1, 0_p, 0_p, 0^{2.1}\}.$$

*Zoneless polar edges :*

$$(55)_{am.gr}^206=2, \quad (44)_{am.gr}^226=4, \quad (33)^246=4.$$

*Zonal edges :*

$$(44)_{zo}26=1, \quad (33)_{zo}46=3, \quad Z=\{2, 2, 0, 0\};$$

$$(44)_{zo}26=3, \quad (33)_{zo}46=4, \quad Z=\{4, 2, 0^3, 0\};$$

$$(44)_{zo}26=1, \quad (33)_{zo}46=1, \quad Z=\{2, 4, 0, 0^3\};$$

$$(44)_{zo}26=2, \quad (33)_{zo}46=2, \quad Z=\{4, 2, 0^2\}.$$

*Epizonal edges :*

$$(55)_{ep}06=1, \quad (37)_{ep}06=1, \quad (35)_{ep}26=2, \quad Z=\{2, 2, 0, 0\};$$

$$(35)_{ep}26=2, \quad Z=\{4, 2, 0^3, 0\};$$

$$(44)_{ep}26=2, \quad (34)_{ep}36=5, \quad Z=\{2, 4, 0, 0^3\};$$

$$(35)_{ep}26=1, \quad (34)_{ep}36=2, \quad (33)_{ep}46=1, \quad Z=\{2, 4, 0^2\}.$$

*Asymmetric edges :*

$$(64)_{as}06=2, \quad (63)_{as}16=10, \quad (54)_{as}16=27;$$

$$(53)_{as}26=65, \quad (44)_{as}26=45;$$

$$(43)_{as}36=169, \quad (33)_{as}46=85.$$

*Registration of 7-edra 10-acra.*

Table A.

1. One 5-zoned monarchaxine, with principal polar pentagons and endrographic secondary axes. This is the pentagonal prism.

2. Two 3-zoned monaxine heteroids, one with polar hexagon and triace, the other with polar triace and triangle. The zones are read in Table C below.

3. One 2-zoned monaxine heteroid, with edrographic axis, carrying a polar tetragon, with the zones

$$Z = \{ \dots, 1_p + 2.1, 0_p, 0^{2.1} \}, \quad Z' = \{ 2.1, 1_p + 2.2, 0_p, 0^{2.2} \}.$$

4. One 2-ple zoneless monaxine heteroid, with edrographic axis terminated by a hexagon.

Table B.

*Janal zoned pole :*

$$\begin{aligned} 5^{5mo} 56 = 1, \quad Z &= \{ 2.1, 1_p + 2.1_{p'}, 0_{p'}, 0^{2.1} \}, \\ Z'' &= \{ \dots, 5.1_{p'}, 0_{p'}^5 \}. \end{aligned}$$

Table C.

*Zoned polar faces :*

$$\begin{aligned} 6^{3ag}_{good} 46 &= 1, \quad Z = \{ 1_p + 1, 1_p + 2, 0, 0^2 \}; \\ (3) 3^{3mo}_{good} 76 &= 1, \quad Z = \{ 1_p + 3, 1_p + 2, 0^2, 0 \}; \\ 4^{2ag}_{edgr} 66 &= 1, \quad Z = \{ \dots, 1_p + 2.1, 0_p, 0^{2.1} \}, \\ Z' &= \{ 2.1, 1_p + 2.2, 0_p, 0^{2.2} \}; \\ 4^{2ag}_{edgr} 66 &= 1, \quad Z = \{ 2.1, 1_p + 2.1, 0_p, 0^{2.1} \}, \\ Z' &= \{ \dots, 1_p + 2.2, 0_p, 0^{2.2} \}. \end{aligned}$$

*Zoneless polar face :*

$$6^2_{edgr} 46 = 1.$$

*Zoned non-polar faces :*

$$\begin{aligned} 5^{mo} 56 &= 1, \quad Z = \{ 2, 3, 0, 0^2 \}; \\ 5^{mo} 56 &= 1, \quad Z = \{ 4, 3, 0^2, 0 \}; \\ 6^{ag} 46 &= 1, \quad Z = \{ \dots, 3, 0^2 \}; \\ 4^{di} 66 &= 1, \quad Z = \{ 4, 3, 0^2, 0 \}; \\ 4^{ag} 66 &= 1, \quad Z = \{ 2, 5, 0, 0^4 \}; \\ 3^{mo} 76 &= 1, \quad Z = \{ 2, 3, 0, 0^2 \}; \\ 3^{mo} 76 &= 1, \quad Z = \{ 2, 5, 0, 0^4 \}. \end{aligned}$$

*Asymmetric faces :*

$$5_{as} 56 = 1, \quad 4_{as} 66 = 1, \quad 3_{as} 76 = 1.$$

Table D.

*Zoned polar edges :*

$$\begin{aligned} (44)^{2a.d}_{edgr} 45 &= 1, \quad Z = \{ 2.1, 1_p + 2.1, 0_p, 0^{2.1} \}, \\ Z' &= \{ \dots, 1_p + 2.2, 0_p, 0^{2.2} \}; \\ (66)^{2a.d}_{edgr} 05 &= 1, \quad Z = \{ 2.1, 1_p + 2.1, 0_p, 0^{2.2} \}, \\ Z' &= \{ \dots, 1_p + 2.1, 0_p, 0^{2.1} \}. \end{aligned}$$

*Zoneless polar edge :*

$$(55)_{edgr}^2 25 = 1.$$

*Zonal edges :*

$$(55)_{zo} 25 = 1, \quad Z = \{2, 3, 0, 0^2\};$$

$$(44)_{zo} 45 = 1, \quad (55)_{zo} 25 = 1, \quad Z = \{4, 3, 0^2, 0\}.$$

*Epizonal edges :*

$$(56)_{ep} 15 = 1, \quad (36)_{ep} 35 = 1, \quad (45)_{ep} 35 = 1, \quad Z = \{2, 3, 0, 0^2\};$$

$$(46)_{ep} 25 = 1, \quad Z = \{., 3, 0^3\};$$

$$(34)_{ep} 55 = 1, \quad Z = \{2, 5, 0, 0^2\}.$$

*Asymmetric edges :*

$$(56)_{as} 15 = 1, \quad (46)_{as} 25 = 2, \quad (36)_{as} 35 = 2;$$

$$(45)_{as} 35 = 3, \quad (35)_{as} 45 = 2, \quad (34)_{as} 55 = 1.$$

*Registration of the 10-edra 7-acra.*

Table A.

1. One 5-zoned monarchaxine, viz. the double pentagonal pyramid.
2. Two 3-zoned monaxine heteroids, one with polar hexace and triangle, the other with polar triace and triangle. The zones are read in order in Table C.

3. One 2-zoned monaxine heteroid, whose gonogrammic axis carries a polar tessarace, with the zones

$$Z = \{1_p + 2.1, \dots, 0_p, 0^{2.1}\}, \quad Z' = \{1_p + 2.2, 2.1, 0_p, 0^{2.2}\}.$$

4. One 2-ple zoneless monaxine heteroid, whose gonogrammic axis carries a hexace.

Table C.

*Zoned polar faces :*

$$^{(5)}3_{go.ed}^{mo} 49 = 1, \quad Z = \{1_p + 2, 1_p + 1, 0^2, 0\};$$

$$^{(4)}3^{mo} 49 = 1, \quad Z = \{1_p + 2, 1_p + 3, 0, 0^2\}.$$

*Zoned non-polar faces :*

$$3^{mo} 49 = 3, \quad Z = \{3, 4, 0, 0^2\};$$

$$3^{mo} 49 = 2, \quad Z = \{3, 2, 0^2, 0\};$$

$$3^{mo} 49 = 1, \quad Z = \{5, 2, 0^4, 0\}.$$

*Asymmetric faces :*

$$3_{as} 49 = 8.$$

Table D.

*Zoned polar edges :*

$$\begin{aligned}
 (33)_{go,gr}^{2a,d}38=1, \quad Z &= \{1_p+2.2, \dots, 0_p, 0^{2.1}\}, \\
 &Z' = \{1_p+2.1, 2.1, 0_p, 0^{2.2}\}; \\
 (33)_{go,gr}^{2a,d}38=1, \quad Z &= \{1_p+2.2, 2.1, 0_p, 0^{2.2}\}, \\
 &Z' = \{1_p+2.1, \dots, 0_p, 0^{2.1}\}.
 \end{aligned}$$

*Zoneless polar edge :*

$$(33)_{go,gr}^238=1.$$

*Zonal edges :*

$$\begin{aligned}
 (33)_{zo}38=3, \quad Z &= \{3, 2, 0^2, 0\}; \\
 (33)_{zo}38=1, \quad Z &= \{3, \dots, 0^3\}; \\
 (33)_{zo}38=2, \quad Z &= \{5, 2, 0^4, 0\}; \\
 (33)_{zo}38=1, \quad Z &= \{3, 4, 0, 0\}.
 \end{aligned}$$

*Epizonal edges :*

$$\begin{aligned}
 (33)_{ep}38=2, \quad Z &= \{3, 4, 0, 0^2\}; \\
 (33)_{ep}38=1, \quad Z &= \{3, 2, 0^2, 0\}.
 \end{aligned}$$

*Asymmetric edges :*

$$(33)_{as}38=11.$$

*Registration of 8-edra 9-acra.*

Table A.

1. One 3-zoned monarchaxine, with principal polar triangles, and gonogrammic secondary axes. The zones are read in Table B.
2. Two 3-zoned monaxine heteroids, one of which has a polar hexagon and triangle, and the other two polar triangles. The zones are the two first read in Table C.
3. One 2-zoned monaxine heteroid, whose gonogrammic axis carries a hexace, with the zones
 
$$Z = \{1_p+2.1, 2.1, 0_p\}, \quad Z' = \{1_p+2.1, 2.1, 0_p, 0^{2.1}\}.$$
4. Five zoneless 2-ple monaxine heteroids, with gonogrammic axes.
5. Seventeen monozones; of which
  - 6 have the zone  $Z = \{3, 2, 0^2, 0\}$ ,
  - 3 have the zone  $Z = \{3, 2, 0\}$ ,
  - 1 has the zone  $Z = \{3, 4, 0^2, 0^3\}$ ,
  - 1 has the zone  $Z = \{5, 2, 0^3\}$ ,
  - 3 have the zone  $Z = \{3, 4, 0, 0^2\}$ ,
  - 3 have the zone  $Z = \{1, 4, 0^3\}$ .
6. Forty-eight asymmetric 8-edra 9-acra.

Table B.

*Zoned janab polar face:*

$$\mathbf{3}_{ja}^{3mo} 6'7 = 1, \quad Z = \{1_p + 2.1, \quad 2_p + 2.1, \quad 0^{2.1}, \quad 0_p, \quad 0^{2.1}\};$$

$$Z'' = 3\{1_p, \quad \dots, \quad 0_p\}.$$

Table C.

*Zoned polar faces:*

$$\mathbf{6}_{amed}^{3ag} 3'7 = 1, \quad Z = \{1, \quad 2_p + 2, \quad 0^3\};$$

$${}^{(4)}\mathbf{3}_{amed}^{3mo} 6'7 = 1, \quad {}^{(3)}\mathbf{3}_{amed}^{3mo} 6'7 = 1, \quad Z = \{3, \quad 2_p + 2, \quad 0, \quad 0^2\};$$

$${}^{(4)}\mathbf{3}_{amed}^{3mo} 6'7 = 1, \quad Z = \{1, \quad 2_p + 2, \quad 0^3\};$$

$${}^{(3)}\mathbf{3}_{amed}^{3mo} 6'7 = 1, \quad Z = \{3, \quad 2_p + 2, \quad 0^2, \quad 0^3\}.$$

*Zoned non-polar faces:*

$$\mathbf{7}^{mo} 2'7 = 2, \quad Z = \{3, \quad 2, \quad 0^2, \quad 0\};$$

$$\mathbf{6}^{ag} 3'7 = 1, \quad Z = \{3, \quad 2, \quad 0^2, \quad 0\};$$

$$\mathbf{6}^{di} 3'7 = 2, \quad Z = \{3, \quad 2, \quad 0\};$$

$$\mathbf{5}^{mo} 4'7 = 2, \quad Z = \{3, \quad 4, \quad 0, \quad 0^2\};$$

$$\mathbf{5}^{mo} 4'7 = 6, \quad Z = \{3, \quad 2, \quad 0^2, \quad 0\};$$

$$\mathbf{5}^{mo} 4'7 = 2, \quad Z = \{1, \quad 4, \quad 0^3\};$$

$$\mathbf{4}^{ag} 5'7 = 3, \quad Z = \{3, \quad 4, \quad 0^2, \quad 0^3\};$$

$$\mathbf{4}^{ag} 5'7 = 2, \quad Z = \{3, \quad 4, \quad 0, \quad 0^2\};$$

$$\mathbf{4}^{ag} 5'7 = 6, \quad Z = \{1, \quad 4, \quad 0^3\};$$

$$\mathbf{4}^{di} 5'7 = 5, \quad Z = \{3, \quad 2, \quad 0\};$$

$$\mathbf{4}^{di} 5'7 = 2, \quad Z = \{3, \quad 4, \quad 0, \quad 0^2\};$$

$$\mathbf{4}^{di} 5'7 = 2, \quad Z = \{5, \quad 2, \quad 0^3\};$$

$$\mathbf{3}^{mo} 6'7 = 2, \quad Z = \{3, \quad 4, \quad 0^2, \quad 0^3\};$$

$$\mathbf{3}^{mo} 6'7 = 8, \quad Z = \{3, \quad 4, \quad 0, \quad 0^2\};$$

$$\mathbf{3}^{mo} 6'7 = 5, \quad Z = \{3, \quad 2, \quad 0^2, \quad 0\};$$

$$\mathbf{3}^{mo} 6'7 = 5, \quad Z = \{1, \quad 4, \quad 0^3\}.$$

*Asymmetric faces:*

$$\mathbf{6}_{as} 3'7 = 13, \quad \mathbf{5}_{as} 4'7 = 67, \quad \mathbf{4}_{as} 5'7 = 156, \quad \mathbf{3}_{as} 6'7 = 213.$$

Table D.

*Zoned polar edge:*

$$(\mathbf{44})_{go,gr}^{2a,d} 36 = 1, \quad Z = \{1_p + 2.1, \quad 2.2, \quad 0_p, \quad 0^{2.1}, \quad 0^{2.1}\},$$

$$Z' = \{1_p + 2.1, \quad \dots, \quad 0_p, \quad 0^{2.1}\}.$$

*Zoneless polar edges :*

$$(55)_{go,gr}^2 16=1, \quad (44)_{go,gr}^2 36=3, \quad (33)_{go,gr}^2 56=1.$$

*Zonal edges :*

$$\begin{aligned} (55)_{zo} 16=2, \quad (44)_{zo} 36=2, \quad Z &= \{3, 4, 0, 0^2\}; \\ (55)_{zo} 16=1, \quad (44)_{zo} 36=6, \quad (33)_{zo} 56=6, \quad Z &= \{3, 2, 0^2, 0\}; \\ (55)_{zo} 16=1, \quad (44)_{zo} 36=1, \quad (33)_{zo} 56=1, \quad Z &= \{3, 4, 0^2, 0^3\}; \\ & (44)_{zo} 36=2, \quad (33)_{zo} 56=1, \quad Z = \{5, 2, 0^3\}; \\ & (44)_{zo} 36=2, \quad (33)_{zo} 56=1, \quad Z = \{3, 2, 0\}. \end{aligned}$$

*Epizonal edges :*

$$\begin{aligned} (64)_{ep} 16=2, \quad (63)_{ep} 26=2, \quad \left. \begin{aligned} (45)_{ep} 26=2, \quad (34)_{ep} 46=4, \end{aligned} \right\} Z &= \{1, 4, 0^3\}; \\ (73)_{ep} 16=2, \quad (55)_{ep} 16=1, \quad Z &= \{3, 2, 0^2, 0\}; \\ (53)_{ep} 36=2, \quad (43)_{ep} 46=4, \quad (33)_{ep} 56=2, \quad Z &= \{3, 4, 0, 0^2\}; \\ & (44)_{ep} 36=1, \quad (33)_{ep} 56=3, \quad Z = \{3, 4, 0^2, 0^3\}. \end{aligned}$$

*Asymmetric edges :*

$$\begin{aligned} (74)_{as} 06=2, \quad (73)_{as} 16=4, \quad (65)_{as} 06=6, \quad (64)_{as} 16=28, \\ (63)_{as} 26=52, \quad (55)_{as} 26=20, \quad (54)_{as} 36=127, \\ (53)_{as} 36=177, \quad (44)_{as} 36=113, \quad (43)_{as} 46=254, \quad (33)_{as} 56=81. \end{aligned}$$

*Registration of 9-edra 8-acra.*

## Table A.

1. One 3-zoned monarchaxine, with principal polar triaces, and edrographic secondary axes. The zones are those first read in Table D below.

2. Two 3-zoned monaxine heteroids, of which one has a polar hexace and triace, and the other two polar triaces. The zones are the reciprocals of the two first read in the above Table C.

3. One 2-zoned monaxine heteroid, whose edrographic axis carries a hexagon, with the zones first written below in Table C.

4. Five zoneless 2-ple monaxine heteroids, with edrographic axes.

5. Seventeen monozones; of which

6 have the zone  $Z = \{2, 3, 0, 0^2\}$ ,

3 have the zone  $Z = \{2, 3, 0\}$ ,

1 has the zone  $Z = \{4, 3, 0^3, 0^2\}$ ,

1 has the zone  $Z = \{2, 5, 0^3\}$ ,

3 have the zone  $Z = \{4, 3, 0^2, 0\}$ ,

3 have the zone  $Z = \{4, 1, 0^3\}$ .

6. Forty-eight asymmetric 9-edra 8-acra.

Table C.

*Zoned polar faces :*

$$\begin{aligned} \underline{6}_{edgr}^{2a.d} 28 &= 1, & Z &= \{2.1, 1_p + 2.1, 0_p\}, \\ & & Z' &= \{2.1, 1_p + 2.1, 0_p, 0^{2.1}\}; \\ 4_{edgr}^{2ag} 48 &= 1, & Z &= \{2.2, 1_p + 2.1, 0_p, 0^{2.1}, 0^{2.1}\}, \\ & & Z' &= \{\dots, 1_p + 2.1, 0_p, 0^{2.1}\}. \end{aligned}$$

*Zoneless polar face :*

$$4_{edgr}^2 48 = 5.$$

*Zoned non-polar faces :*

$$\begin{aligned} 6^{ag} 28 &= 1, & Z &= \{\dots, 3, 0^3\}; \\ 6^{di} 28 &= 1, & Z &= \{4, 1, 0^3\}; \\ 5^{mo} 38 &= 2, & Z &= \{2, 3, 0\}; \\ 5^{mo} 38 &= 3, & Z &= \{2, 3, 0, 0^2\}; \\ 4^{di} 48 &= 3, & Z &= \{2, 3, 0\}; \\ 4^{di} 48 &= 4, & Z &= \{4, 3, 0^2, 0\}; \\ 4^{di} 48 &= 3, & Z &= \{4, 1, 0^3\}; \\ 4^{ag} 48 &= 5, & Z &= \{2, 3, 0, 0^2\}; \\ 4^{ag} 48 &= 1, & Z &= \{4, 3, 0^3, 0^2\}; \\ 4^{ag} 48 &= 1, & Z &= \{2, 5, 0^3\}; \\ 3^{mo} 58 &= 5, & Z &= \{2, 3, 0\}; \\ 3^{mo} 58 &= 10, & Z &= \{2, 3, 0, 0^2\}; \\ 3^{mo} 58 &= 3, & Z &= \{4, 3, 0^3, 0^2\}; \\ 3^{mo} 58 &= 8, & Z &= \{4, 3, 0^2, 0\}; \\ 3^{mo} 58 &= 4, & Z &= \{2, 5, 0^3\}. \end{aligned}$$

*Asymmetric faces :*

$$5_{as} 38 = 19, \quad 4_{as} 48 = 121, \quad 3_{as} 58 = 367.$$

Table D.

*Zoned polar edges :*

$$\begin{aligned} (44)_{ed.gr}^{2a.d} 27 &= 1, & Z &= \{2.2, 1_p + 2.1, 0_p, 0^{2.1}, 0^{2.1}\}, \\ & & Z' &= \{\dots, 1_p + 2.1, 0_p, 0^{2.1}\}; \\ (33)_{ed.gr}^{2a.d} 47 &= 1, & Z &= \{2.1, 1_p + 2.1, 0_p, 0^{2.1}\}, \\ & & Z' &= \{2.1, 1_p + 2.1, 0_p\}. \end{aligned}$$

*Zoneless polar edges :*

$$(44)_{ed.gr}^2 27 = 2, \quad (33)_{ed.gr}^2 47 = 3.$$

*Zonal edges:*

$$\begin{aligned}
 (44)_{zo}2^7 &= 4, & (33)_{zo}4^7 &= 4, & Z &= \{4, 3, 0^2, 0\}; \\
 (44)_{zo}2^7 &= 3, & (33)_{zo}4^7 &= 9, & Z &= \{4, 1, 0^3\}; \\
 (44)_{zo}2^7 &= 1, & (33)_{zo}4^7 &= 5, & Z &= \{2, 3, 0, 0^2\}; \\
 (44)_{zo}2^7 &= 1, & (33)_{zo}4^7 &= 3, & Z &= \{4, 3, 0^2, 0\}.
 \end{aligned}$$

*Epizonal edges:*

$$\begin{aligned}
 (63)_{ep}1^7 &= 3, & (54)_{ep}1^7 &= 3, & (43)_{ep}3^7 &= 7, & Z &= \{2, 3, 0, 0^2\}; \\
 (53)_{ep}2^7 &= 2, & (33)_{ep}4^7 &= 1, & & & Z &= \{2, 3, 0\}; \\
 (43)_{ep}3^7 &= 3, & & & & & Z &= \{4, 3, 0^3, 0^2\}; \\
 (33)_{ep}4^7 &= 4, & & & & & Z &= \{4, 3, 0^2, 0\}; \\
 (43)_{ep}3^7 &= 2, & (33)_{ep}4^7 &= 1, & & & Z &= \{2, 5, 0^3\}.
 \end{aligned}$$

*Asymmetric edges:*

$$\begin{aligned}
 (63)_{as}1^7 &= 6, & (53)_{as}2^7 &= 90, & (54)_{as}1^7 &= 15; \\
 (44)_{as}2^7 &= 62, & (43)_{as}3^7 &= 371, & (33)_{as}4^7 &= 320.
 \end{aligned}$$

*Registration of 10-edra 8-acra.*

## Table A.

1. One 4-zoned monarchaxine homozone, with principal polar tetragons, and amphigrammic zoneless axes. The zone is first read in Table B below.

2. One homozone triaxine, with zoned tetragon poles, and amphigrammic zoneless axes, with zone next read in Table B.

3. One 2-ple monaxine monozone, with amphigrammic axis, with the zone  $Z = \{2, 2, 2, 1, 0^{2,1}\}$ .

4. Two 2-zoned monaxine heteroids, one with amphigrammic axis, having the zones

$$Z = \{2, 1, 2, 2, 0_p, 0_p, 0^{2,1}\}, \quad Z' = \{2, 1, 2, 1, 0_p, 0_p\},$$

the other with amphigonal axis, carrying a hexace and a tesseract, with the zones

$$Z = \{2, 1_p, 2, 2, 0^{2,1}\}, \quad Z' = \{2, 1_p + 2, 1, 2, 1, 0^{2,1}\}.$$

5. Eight zoneless 2-ple monaxine heteroids, one having an amphigonal axis, and seven with amphigrammic axes.

6. Nineteen monozones, of which

$$6 \text{ have the zone } Z = \{4, 2, 0^3, 0\},$$

$$1 \text{ has the zone } Z = \{4, 2, 0^2\},$$

$$2 \text{ have the zone } Z = \{4, 4, 0^2, 0^2\},$$

$$3 \text{ have the zone } Z = \{2, 4, 0^2\},$$

2 have the zone  $Z = \{2, 4, 0, 0^3\}$ ,

4 have the zone  $Z = \{2, 2, 0, 0\}$ ,

1 has the zone  $Z = \{4, \dots, 0^4\}$ .

7. Forty-four asymmetric 10-edra 8-acra.

Table B.

*Homozone janal polar faces:*

$$4_{ja.hom}^{4a.d} 49 = 1, \quad Z = \{2.1, 2_p + 2.1, 0^{2.1}\}, \\ \zeta = 8\{0_p\};$$

$$4_{ja.hom}^{2di} 49 = 1, \quad Z = \{2.2, 2_p, 0^{2.1}\}, \\ \zeta = 4\{0_p\}.$$

*Janal zoneless amphigrammic poles:*

$$(33)_{ja}^2 48 = 2, \quad (33)_{co,ja}^2 48 = 1.$$

Table C.

*Zoned non-polar faces:*

$$5^{mo} 39 = 3, \quad Z = \{2, 2, 0, 0\};$$

$$5^{mo} 39 = 3, \quad Z = \{4, 2, 0^3, 0\};$$

$$5^m 39 = 2, \quad Z = \{2, 4, 0^2\};$$

$$4^{di} 49 = 4, \quad Z = \{4, 2, 0^2\};$$

$$4^{di} 49 = 1, \quad Z = \{2, 4, 0^2\};$$

$$4^{ag} 49 = 5, \quad Z = \{2, 4, 0, 0^3\};$$

$$4^{ag} 49 = 1, \quad Z = \{2, 4, 0^2\};$$

$$3^{mo} 59 = 9, \quad Z = \{4, 2, 0^3, 0\};$$

$$3^{mo} 59 = 6, \quad Z = \{2, 2, 0, 0\};$$

$$3^{mo} 59 = 8, \quad Z = \{4, 4, 0_2, 0\};$$

$$3^{mo} 59 = 9, \quad Z = \{2, 4, 0^2\}.$$

*Objanal monozone face:*

$$4_{obj}^{di} 49 = 1, \quad Z = \{2.2, 2.1, 0^{2.1}\}.$$

*Asymmetric faces:*

$$5_{as} 39 = 4, \quad 4_{as} 49 = 95, \quad 3_{as} 59 = 456.$$

*Janal anaxine faces:*

$$3_{ja.an} 59 = 2.$$

Table D.

*Zoned polar edges:*

$$(33)_{am.gr}^{2a.d} 48 = 1, \quad (44)_{am.gr}^{2a.d} 28 = 1, \quad Z = \{2.1, 2.2, 0_p, 0_p, 0^{2.1}\}, \\ Z' = \{2.1, 2.1, 0_p, 0_p\}.$$

*Zoneless polar edges :*

$$(33)_{am,gr}^2 48 = 12, \quad (44)_{am,gr}^2 28 = 5.$$

*Objanal zonal edge :*

$$(33)_{zo,obj} 48 = 1, \quad Z = \{2.2, 2.1, 0^{2.1}\}.$$

This edge is also enumerated below among the zonals of the signature  $\{4, 2, 0^2\}$  (*vide* note to art. XLIX.).

*Zonal non-polar edges :*

$$\begin{aligned} (44)_{zo} 28 &= 1, & (33)_{zo} 48 &= 3, & Z &= \{4, \dots, 0^4\}; \\ (44)_{zo} 28 &= 3, & (33)_{zo} 48 &= 15, & Z &= \{4, 2, 0^3, 0^2\}; \\ (44)_{zo} 28 &= 2, & (33)_{zo} 48 &= 2, & Z &= \{4, 4, 0^2, 0^2\}; \\ (33)_{zo} 48 &= 4, & & & Z &= \{2, 2, 0, 0\}; \\ (33)_{zo} 48 &= 2, & & & Z &= \{2, 4, 0, 0^3\}; \\ (33)_{zo} 48 &= 5, & & & Z &= \{4, 2, 0^2\}. \end{aligned}$$

*Epizonal edges :*

$$\begin{aligned} (53)_{ep} 28 &= 3, & (33)_{ep} 48 &= 1, & Z &= \{2, 2, 0, 0\}; \\ (53)_{ep} 28 &= 3, & (33)_{ep} 48 &= 3, & Z &= \{4, 2, 0^3, 0\}; \\ (44)_{ep} 28 &= 2, & (34)_{ep} 38 &= 4, & Z &= \{2, 4, 0, 0^3\}; \\ (53)_{ep} 28 &= 2, & (33)_{ep} 48 &= 3, & Z &= \{2, 4, 0^2\}; \\ (33)_{ep} 48 &= 4, & & & Z &= \{4, 4, 0^2, 0^2\}. \end{aligned}$$

*Asymmetric edges :*

$$\begin{aligned} (53)_{as} 28 &= 36, & (44)_{as} 28 &= 22; \\ (34)_{as} 38 &= 342, & (33)_{as} 48 &= 493. \end{aligned}$$

*Janal anaxine edges :*

$$(33)_{ja,an} 48 = 2, \quad (43)_{ja,an} 38 = 1.$$

### *Registration of 3-edra 10-acra.*

Table A.

1. One 4-zoned monarchaxine homozone, with principal polar tessaraces, and amphigrammic zoneless axes. The zone is

$$Z = \{2_p + 2.1, 2.1, 0^{2.1}\}.$$

2. One homozone triaxine, with zoned polar tessaraces, and amphigrammic zoneless axes. The zone is  $Z = \{2_p, 2.2, 0^{2.1}\}$ .

3. One 2-ple monaxine monozone, with amphigrammic axis. The zone is  $Z = \{2.1, 2.2, 0^{2.1}\}$ .

4. Two 2-zoned monaxine heteroids, one with amphigrammic axis, having the zones

$$Z = \{2.2, 2.1, 0_p, 0_p, 0^{2.1}\}, \quad Z' = \{2.1, 2.1, 0_p, 0_p\},$$

the other having an amphidral axis, with the zones

$$Z = \{2.2, 2.1_p, 0^{2.1}\}, \quad Z' = \{2.1, 2.1_p + 2.1, 0^{2.1}\}.$$

5. Eight zoneless 2-ple monaxine heteroids, one having an amphidral axis, and seven with amphigrammic axes.

6. Nineteen monozones, of which

$$6 \text{ have the zone } Z = \{2, 4, 0, 0^3\},$$

$$1 \text{ has the zone } Z = \{2, 4, 0^2\},$$

$$2 \text{ have the zone } Z = \{4, 4, 0^2, 0^2\},$$

$$3 \text{ have the zone } Z = \{4, 2, 0^2\},$$

$$2 \text{ have the zone } Z = \{4, 2, 0^3, 0\},$$

$$4 \text{ have the zone } Z = \{2, 2, 0, 0\},$$

$$1 \text{ has the zone } Z = \{.., 4, 0^4\}.$$

7. Forty-four asymmetric 8-edra 10-acra.

Table B.

*Janal zoneless amphigrammic poles :*

$$(55)_{ja}^2 26 = 1, \quad (44)_{ja}^2 46 = 1, \quad (44)_{co,ja}^2 46 = 1.$$

Table C.

*Zoned polar faces :*

$$\left. \begin{array}{l} 6_{am,ed}^{2a,d} 47 = 1, \\ 4_{am,ed}^{2,di} 67 = 1, \end{array} \right\} \left\{ \begin{array}{l} Z = \{2.2, 2_p, 0^{2.1}\}, \\ Z' = \{2.1, 2_p + 2.1, 0^{2.1}\}. \end{array} \right.$$

*Zoneless polar face :*

$$4_{am,ed}^2 67 = 2.$$

*Zoned non-polar faces :*

$$7^{mo} 37 = 1, \quad Z = \{4, 2, 0^3, 0\};$$

$$7^{mo} 37 = 3, \quad Z = \{2, 2, 0, 0\};$$

$$6^{ag} 47 = 1, \quad Z = \{.., 4, 0^4\};$$

$$6^{ag} 47 = 3, \quad Z = \{2, 4, 0, 0^3\};$$

$$6^{di} 47 = 2, \quad Z = \{4, 2, 0^2\};$$

$$5^{mo} 57 = 1, \quad Z = \{4, 4, 0^2, 0^2\};$$

$$5^{mo} 57 = 2, \quad Z = \{4, 2, 0^3\};$$

$$5^{mo} 57 = 3, \quad Z = \{2, 4, 0, 0^3\};$$

$$5^{mo} 57 = 3, \quad Z = \{2, 4, 0^2\};$$

$$4^{di} 67 = 5, \quad Z = \{4, 2, 0^2\};$$

$$4^{di} 67 = 1, \quad Z = \{4, 4, 0^2, 0^2\};$$

$$4^{di} 67 = 1, \quad Z = \{2, 4, 0^2\};$$

$$\begin{aligned}
4^{ag}67 &= 1, & Z &= \{2, 4, 0^2\}; \\
4^{ag}67 &= 9, & Z &= \{2, 4, 0, 0^3\}; \\
4^{ag}67 &= 3, & Z &= \{., 4, 0^4\}; \\
4^{ag}67 &= 1, & Z &= \{4, 4, 0^2, 0^2\}; \\
3^{mo}77 &= 2, & Z &= \{4, 2, 0^3, 0\}; \\
3^{mo}77 &= 9, & Z &= \{2, 4, 0, 0^3\}; \\
3^{mo}77 &= 4, & Z &= \{2, 4, 0^2\}; \\
3^{mo}77 &= 5, & Z &= \{4, 4, 0^2, 0^2\}; \\
3^{mo}77 &= 1, & Z &= \{2, 2, 0, 0\}.
\end{aligned}$$

*Objanal monozone faces :*

$$5_{obj}^{mo}57 = 1, \quad 3_{obj}^{mo}77 = 1, \quad Z = \{2.1, 2.2, 0^{2.1}\}.$$

These are also entered above.

*Janal anaxine face :*

$$4_{ja.an}67 = 1, \text{ which is also entered below.}$$

*Asymmetric faces :*

$$\begin{aligned}
7_{as}37 &= 2, & 6_{as}47 &= 30; \\
5_{as}57 &= 90, & 4_{as}67 &= 146, & 3_{as}77 &= 165.
\end{aligned}$$

Table D.

*Zoned polar edges :*

$$\begin{aligned}
(55)_{am.gr}^{2a.d}26 &= 2, & Z &= \{2.2, 2.1, 0_p, 0_p, 0^{2.1}\}, \\
& & Z' &= \{2.1, 2.1, 0_p, 0_p\}.
\end{aligned}$$

*Zoneless polar edges :*

$$\begin{aligned}
(66)_{am.gr}^206 &= 2, & (55)_{am.gr}^226 &= 5; \\
(33)_{am.gr}^266 &= 2, & (44)_{am.gr}^246 &= 8.
\end{aligned}$$

*Zonal non-polar edges :*

$$\begin{aligned}
(55)_{zo}26 &= 1, & (44)_{zo}46 &= 4, & (33)_{zo}66 &= 2, & Z &= \{4, 2, 0^2\}; \\
(55)_{zo}26 &= 3, & (44)_{zo}46 &= 2, & (33)_{zo}66 &= 1, & Z &= \{2, 4, 0, 0^3\}; \\
(55)_{zo}26 &= 2, & (44)_{zo}46 &= 2, & & & Z &= \{4, 4, 0^2, 0^2\}; \\
(55)_{zo}26 &= 1, & (44)_{zo}46 &= 3, & (33)_{zo}66 &= 3, & Z &= \{4, 2, 0^3, 0\}; \\
(44)_{zo}46 &= 3, & (33)_{zo}66 &= 1, & & & Z &= \{2, 2, 0, 0\}.
\end{aligned}$$

*Epizonal edges :*

$$\begin{aligned}
(57)_{ep}06 &= 2, & (55)_{ep}26 &= 1, & (37)_{ep}26 &= 1, & Z &= \{2, 2, 0, 0\}; \\
(53)_{ep}46 &= 1, & (43)_{ep}56 &= 2, & (33)_{ep}66 &= 1, & Z &= \{4, 2, 0^2\}; \\
(73)_{ep}26 &= 1, & (53)_{ep}46 &= 1, & & & Z &= \{4, 2, 0^3, 0\};
\end{aligned}$$

$$\begin{aligned}
 (63)_{ep}36=1, \quad (54)_{ep}36=1, \quad (53)_{ep}46=2, \quad (43)_{ep}56=1, \quad Z=\{2, 4, 0^2\}; \\
 (63)_{ep}36=3, \quad (54)_{ep}36=3, \quad (44)_{ep}46=3, \quad \left. \begin{array}{l} (64)_{ep}26=3, \quad (34)_{ep}56=6, \end{array} \right\} Z=\{2, 4, 0, 0^3\}; \\
 (53)_{ep}46=1, \quad (43)_{ep}56=2, \quad (33)_{ep}66=1, \quad Z=\{4, 4, 0^2, 0^2\}.
 \end{aligned}$$

*Asymmetric edges :*

$$\begin{aligned}
 (75)_{as}06=2, \quad (66)_{as}06=2, \quad (74)_{as}16=10; \\
 (65)_{as}16=32, \quad (73)_{as}26=14, \quad (64)_{as}26=69; \\
 (55)_{as}26=44, \quad (63)_{as}36=88, \quad (54)_{as}36=169; \\
 (53)_{as}46=174, \quad (44)_{as}46=91, \quad (43)_{as}56=165, \quad (33)_{as}66=33.
 \end{aligned}$$

*Janal anaxine edges :*

$$(54)_{ja.an}36=2, \quad (43)_{ja.an}56=1.$$

### *Registration of 9-edra 9-acra.*

Table A.

1. One 8-zoned monaxine heteroid, with gonoedral axis, viz. the octagonal pyramid.

2. Two 4-zoned monaxine heteroids, with gonoedral axes, each carrying a tesseract and a tetragon, with zones below read in Table C.

3. Eight zoneless 2-ple monaxine heteroids, with gonoedral axes.

4. Forty-eight monozone, of which

$$\begin{array}{lll}
 11 & \text{have the zone } Z=\{3, \mathbf{3}, 0^2, 0^2\}, \\
 15 & \text{,,} \quad Z=\{3, \mathbf{3}, 0, 0\}, \\
 7 & \text{,,} \quad Z=\{1, \mathbf{3}, 0^2\}, \\
 7 & \text{,,} \quad Z=\{3, \mathbf{1}, 0^2\}, \\
 2 & \text{,,} \quad Z=\{3, \mathbf{5}, 0, 0^3\}, \\
 2 & \text{,,} \quad Z=\{5, \mathbf{3}, 0^3, 0\}, \\
 2 & \text{,,} \quad Z=\{1, \mathbf{5}, 0^4\}, \\
 2 & \text{,,} \quad Z=\{5, \mathbf{1}, 0^4\}.
 \end{array}$$

5. Two hundred and thirty-seven asymmetrical 9-edra 9-acra.

*Zoned polar faces :*

Table C.

$$\begin{aligned}
 8_{go.ed}^{8a.d}18=1, \quad Z=\{3, \mathbf{1}, 0^3\}, \quad Z'=\{1, \mathbf{3}, 0^3\}; \\
 4_{go.ed}^{4a.d}58=1, \quad Z=\{1_p+2.1, \mathbf{1}_p+2.1\}, \\
 \quad Z'=\{1_p+2.1, \mathbf{1}_p+2.1, 0^{2.1}, 0^{2.1}\}; \\
 4_{go.ed}^{4a.d}58=1, \quad Z=\{1_p, \mathbf{1}_p+2.2, 0^{2.2}\}, \\
 \quad Z'=\{1_p+2.2, \mathbf{1}_p, 0^{2.2}\}.
 \end{aligned}$$

*Zoneless polar faces :*

$$6_{go.ed}^2 38=2, \quad 4_{go.ed}^2 58=6.$$

*Zoned non-polar faces ;*

$$\begin{aligned} 7^{mo} 28 &= 1, & Z &= \{1, 3, 0^2\}; \\ 6^{ag} 38 &= 3, & Z &= \{1, 3, 0^2\}; \\ 6^{ag} 38 &= 3, & Z &= \{3, 3, 0^2, 0^2\}; \\ 6^{di} 38 &= 2, & Z &= \{3, 3, 0, 0\}; \\ 6^{di} 38 &= 3, & Z &= \{3, 1, 0^2\}; \\ 6^{di} 38 &= 1, & Z &= \{5, 1, 0^4\}; \\ 5^{mo} 48 &= 6, & Z &= \{1, 3, 0^2\}; \\ 5^{mo} 48 &= 10, & Z &= \{3, 3, 0, 0\}; \\ 5^{mo} 48 &= 6, & Z &= \{3, 3, 0^2, 0^2\}; \\ 4^{ag} 58 &= 4, & Z &= \{1, 3, 0^2\}; \\ 4^{ag} 58 &= 8, & Z &= \{3, 3, 0^2, 0^2\}; \\ 4^{ag} 58 &= 2, & Z &= \{3, 5, 0, 0^3\}; \\ 4^{ag} 58 &= 7, & Z &= \{1, 5, 0^4\}; \\ 4^{di} 58 &= 13, & Z &= \{3, 3, 0, 0\}; \\ 4^{di} 58 &= 1, & Z &= \{3, 3\}; \\ 4^{di} 58 &= 4, & Z &= \{1, 3, 0^2\}; \\ 4^{di} 58 &= 1, & Z &= \{5, 1, 0^4\}; \\ 4^{di} 58 &= 2, & Z &= \{5, 3, 0^3, 0\}; \\ 3^{mo} 68 &= 17, & Z &= \{3, 3, 0^2, 0^2\}; \\ 3^{mo} 68 &= 20, & Z &= \{3, 3, 0, 0\}; \\ 3^{mo} 68 &= 8, & Z &= \{1, 3, 0^2\}; \\ 3^{mo} 68 &= 4, & Z &= \{5, 3, 0^2, 0\}; \\ 3^{mo} 68 &= 8, & Z &= \{3, 5, 0, 0^3\}; \\ 3^{mo} 68 &= 5, & Z &= \{1, 5, 0^4\}. \end{aligned}$$

*Asymmetric faces :*

$$\begin{aligned} 7_{as} 28 &= 2, & 6_{as} 38 &= 30, \\ 5_{as} 48 &= 221, & 4_{as} 58 &= 717, & 3_{as} 68 &= 1344. \end{aligned}$$

The summits of the 9-edra 9-acra are the reciprocals of the foregoing faces.

Table D.

*Zonal non-polar edges :*

|                         |                         |                          |                          |
|-------------------------|-------------------------|--------------------------|--------------------------|
| $(55)_{zo}1\bar{7}=1$ , | $(44)_{zo}3\bar{7}=6$ , | $(33)_{zo}5\bar{7}=8$ ,  | $Z=\{3, 1, 0^2\}$ ;      |
| $(55)_{zo}1\bar{7}=1$ , | $(44)_{zo}3\bar{7}=1$ , |                          | $Z=\{3, 5, 0, 0^3\}$ ;   |
| $(55)_{zo}1\bar{7}=1$ , | $(44)_{zo}3\bar{7}=9$ , | $(33)_{zo}5\bar{7}=13$ , | $Z=\{3, 3, 0^2, 0^2\}$ ; |
| $(55)_{zo}1\bar{7}=1$ , | $(44)_{zo}3\bar{7}=8$ , | $(33)_{zo}5\bar{7}=6$ ,  | $Z=\{3, 3, 0, 0\}$ ;     |
|                         | $(44)_{zo}3\bar{7}=4$ , | $(33)_{zo}5\bar{7}=6$ ,  | $Z=\{5, 1, 0^4\}$ ;      |
|                         | $(44)_{zo}3\bar{7}=4$ , | $(33)_{zo}5\bar{7}=2$ ,  | $Z=\{5, 3, 0^3, 0\}$ .   |

*Epizonal edges :*

|                          |                          |                         |                          |
|--------------------------|--------------------------|-------------------------|--------------------------|
| $(54)_{ep}2\bar{7}=6$ ,  | $(43)_{ep}4\bar{7}=11$ , | $(63)_{ep}2\bar{7}=6$ , | $Z=\{3, 3, 0^2, 0^2\}$ ; |
| $(53)_{ep}3\bar{7}=10$ , | $(33)_{ep}5\bar{7}=5$ ,  |                         | $Z=\{3, 3, 0^2, 0^2\}$ ; |
| $(56)_{ep}0\bar{7}=2$ ,  | $(54)_{ep}2\bar{7}=4$ ,  | $(63)_{ep}2\bar{7}=4$ , | $Z=\{1, 3, 0^2\}$ ;      |
| $(83)_{ep}0\bar{7}=1$ ,  | $(74)_{ep}0\bar{7}=1$ ,  | $(43)_{ep}4\bar{7}=3$ , |                          |
| $(43)_{ep}4\bar{7}=4$ ,  | $(33)_{ep}5\bar{7}=2$ ,  |                         | $Z=\{3, 5, 0, 0^3\}$ ;   |
|                          | $(33)_{ep}5\bar{7}=2$ ,  |                         | $Z=\{5, 3, 0^3, 0\}$ ;   |
| $(44)_{ep}3\bar{7}=5$ ,  | $(43)_{ep}4\bar{7}=5$ ,  |                         | $Z=\{1, 5, 0^4\}$ .      |

*Asymmetric edges :*

|                            |                           |                           |
|----------------------------|---------------------------|---------------------------|
| $(73)_{as}1\bar{7}=15$ ,   | $(74)_{as}0\bar{7}=2$ ,   | $(63)_{as}2\bar{7}=158$ ; |
| $(64)_{as}1\bar{7}=54$ ,   | $(65)_{as}0\bar{7}=5$ ,   | $(55)_{as}1\bar{7}=38$ ;  |
| $(54)_{as}2\bar{7}=339$ ,  | $(53)_{as}3\bar{7}=719$ , | $(44)_{as}3\bar{7}=487$ ; |
| $(43)_{as}4\bar{7}=1532$ , | $(33)_{as}5\bar{7}=808$ . |                           |

*Registration of the 8-edra 11-acra.*

Table A.

1. Four 2-zoned monaxine heteroids, with gonogrammic axes carrying tessaraces, with the zones first read below in Table D.

2. One 2-ple zoneless monaxine heteroid, with gonogrammic axis carrying a tessarace.

3. Twelve monozones, of which

- 4 have the zone  $Z=\{3, 4, 0, 0^2\}$ ,
- 2 have the zone  $Z=\{3, 4, 0^2, 0^3\}$ ,
- 2 have the zone  $Z=\{3, 2, 0^2, 0\}$ ,
- 1 has the zone  $Z=\{3, 2, 0\}$ ,
- 1 has the zone  $Z=\{5, 2, 0^3\}$ ,
- 2 have the zone  $Z=\{1, 4, 0^3\}$ .

4. Twenty-one asymmetric 8-edra 11-acra.

*Zoned non-polar faces:*

Table C.

|                |                          |
|----------------|--------------------------|
| $7^{mo}47=2$ , | $Z=\{3, 2, 0^2, 0\}$ ;   |
| $6^{di}57=2$ , | $Z=\{3, 2, 0^2, 0\}$ ;   |
| $6^{di}57=1$ , | $Z=\{5, 2, 0^3\}$ ;      |
| $6^{ag}57=1$ , | $Z=\{3, 4, 0, 0^2\}$ ;   |
| $6^{ag}57=1$ , | $Z=\{3, 4, 0^2, 0^2\}$ ; |
| $6^{ag}57=3$ , | $Z=\{1, 4, 0^3\}$ ;      |
| $5^{mo}67=3$ , | $Z=\{1, 4, 0^3\}$ ;      |
| $5^{mo}67=6$ , | $Z=\{3, 4, 0, 0^2\}$ ;   |
| $5^{mo}67=1$ , | $Z=\{3, 4, 0^2, 0^3\}$ ; |
| $5^{mo}67=2$ , | $Z=\{3, 2, 0^2, 0\}$ ;   |
| $4^{di}77=1$ , | $Z=\{3, 2, 0^2, 0\}$ ;   |
| $4^{di}77=2$ , | $Z=\{5, 2, 0^3\}$ ;      |
| $4^{di}77=3$ , | $Z=\{3, 4, 0, 0^2\}$ ;   |
| $4^{ag}77=3$ , | $Z=\{1, 4, 0^3\}$ ;      |
| $4^{ag}77=2$ , | $Z=\{1, 6, 0^5\}$ ;      |
| $4^{ag}77=3$ , | $Z=\{3, 4, 0^2, 0^3\}$ ; |
| $4^{ag}77=2$ , | $Z=\{3, 4, 0, 0^2\}$ ;   |
| $3^{mo}87=3$ , | $Z=\{1, 4, 0^3\}$ ;      |
| $3^{mo}87=1$ , | $Z=\{1, 6, 0^5\}$ ;      |
| $3^{mo}87=1$ , | $Z=\{3, 2, 0^2, 0\}$ ;   |
| $3^{mo}87=8$ , | $Z=\{3, 4, 0, 0^2\}$ ;   |
| $3^{mo}87=3$ , | $Z=\{3, 4, 0^2, 0^3\}$ . |

*Asymmetric faces:*

|                |                 |                 |
|----------------|-----------------|-----------------|
| $7_{as}47=5$ , | $6_{as}57=24$ , | $5_{as}67=47$ ; |
|                | $4_{as}77=63$ , | $3_{as}87=62$ . |

*Zoned polar edges:*

Table D.

|                             |                                       |
|-----------------------------|---------------------------------------|
| $(66)_{go.gr}^{2a.d}16=1$ , | $Z=\{1_p+2.1, 2.2, 0_p, 0^{2.1}\}$ ,  |
|                             | $Z'=\{1_p, 2.2, 0_p, 0^{2.1}\}$ ;     |
| $(44)_{go.gr}^{2a.d}56=1$ , | $Z=\{1_p+2.1, 2.2, 0_p, 0^{2.1}\}$ ,  |
|                             | $Z'=\{1_p, 2.2, 0_p, 0^{2.1}\}$ ;     |
| $(55)_{go.gr}^{2a.d}36=1$ , | $Z=\{1_p+2.2, 2.1, 0_p, 0^{2.1}\}$ ,  |
|                             | $Z'=\{1_p+2.1, 2.1, 0_p, 0^{2.1}\}$ ; |
| $(44)_{go.gr}^{2a.d}56=1$ , | $Z=\{1_p+2.1, 2.1, 0_p\}$ ,           |
|                             | $Z'=\{1_p, 2.3, 0_p, 0^{2.2}\}$ .     |

*Zoneless polar edge:*

$$(55)_{90,9r}^2 \quad 36 = 1.$$

*Zonal non-polar edges:*

$$\begin{array}{llll}
 (66)_{zo}16=1, & (55)_{zo}36=2, & (44)_{zo}56=1, & Z=\{3, 4, 0, 0^2\}; \\
 (44)_{zo}56=1, & (66)_{zo}16=1, & & \\
 (55)_{zo}36=1, & (33)_{zo}76=1, & & Z=\{3, 4, 0^2, 0^3\}; \\
 (55)_{zo}36=1, & (44)_{zo}56=2, & (33)_{zo}76=1, & Z=\{5, 2, 0^3\}; \\
 (55)_{zo}36=2, & (44)_{zo}56=2, & (33)_{zo}76=1, & Z=\{3, 2, 0^2, 0\}; \\
 (55)_{zo}36=1, & & & Z=\{3, 2, 0\}.
 \end{array}$$

*Epizonal edges :*

$$\begin{array}{lll}
(53)_{ep}56=4, & (54)_{ep}46=2, & \\
(63)_{ep}46=2, & (43)_{ep}66=2, & \left. \begin{array}{l} \\ \end{array} \right\} Z=\{3, 4, 0, 0^2\}; \\
(63)_{ep}46=1, & (54)_{ep}46=1, & (64)_{ep}36=1, \left. \begin{array}{l} \\ \end{array} \right\} Z=\{3, 4, 0^2, 0^3\}; \\
(44)_{ep}56=1, & (43)_{ep}66=2, & \\
(75)_{ep}16=1, & (73)_{ep}36=1, & Z=\{3, 2, 0^2, 0\}; \\
(65)_{ep}26=1, & (64)_{ep}36=2, & (63)_{ep}46=2, \left. \begin{array}{l} \\ \end{array} \right\} Z=\{1, 4, 0^3\}; \\
(54)_{ep}46=2, & (34)_{ep}66=1, & \\
(44)_{ep}56=1, & (34)_{ep}66=1, & Z=\{1, 6, 0^5\}.
\end{array}$$

*Asymmetric edges :*

$$\begin{array}{lll} (73)_{as}36=17, & (74)_{as}26=15, & (76)_{as}06=2; \\ (66)_{as}16=6, & (65)_{as}26=40, & (64)_{as}36=53; \\ (63)_{as}36=54, & (55)_{as}36=27, & (54)_{as}46=82; \\ (43)_{as}66=50, & (53)_{as}56=68, & (44)_{as}56=34, & (33)_{as}76=5. \end{array}$$

*Registration of 11-edra 8-acra.*

1. Four 2-zoned monaxine heteroids, with edrographic axes, carrying tetragons and zones below written in Table C.
2. One 2-ple zoneless monaxine heteroid, with edrographic axis, carrying a tetragon.
3. Twelve monozones, of which
  - 4 have the zone  $Z = \{4, \mathbf{3}, \mathbf{0}^2, 0\}$ ,
  - 2 have the zone  $Z = \{4, \mathbf{3}, \mathbf{0}^3, 0^2\}$ ,
  - 2 have the zone  $Z = \{2, \mathbf{3}, \mathbf{0}, 0^2\}$ ,
  - 1 has the zone  $Z = \{2, \mathbf{3}, 0\}$ ,
  - 1 has the zone  $Z = \{2, \mathbf{5}, 0^3\}$ ,
  - 2 have the zone  $Z = \{4, \mathbf{1}, \mathbf{0}^3\}$ .
4. Twenty-one asymmetric 11-edra 8-acra.

Table C.

*Zoned polar faces :*

$$4_{ed.gr}^{2di}410=1, \quad Z=\{2.3, 1_p, 0_p, 0^{2.2}\},$$

$$Z'=\{2.1, 1_p+2.1, 0_p\};$$

$$4_{ed.gr}^{2di}410=2, \quad Z=\{2.2, 1_p+2.1, 0_p, 0^{2.1}\},$$

$$Z'=\{2.2, 1_p, 0_p, 0^{2.1}\};$$

$$4_{ed.gr}^{2ag}410=1, \quad Z=\{2.1, 1_p+2.2, 0_p, 0^{2.1}\},$$

$$Z'=\{2.1, 1_p+2.1, 0_p, 0^{2.1}\}.$$

*Zoneless polar face :*

$$4_{ed.gr}^2410=1.$$

*Zoned non-polar faces :*

$$4^{di}410=2, \quad Z=\{4, 1, 0^3\};$$

$$4^{di}410=4, \quad Z=\{4, 3, 0^2, 0\};$$

$$4^{ag}410=2, \quad Z=\{4, 3, 0^3, 0^2\};$$

$$4^{ag}410=2, \quad Z=\{2, 3, 0, 0^2\};$$

$$4^{di}410=1, \quad Z=\{2, 3, 0\};$$

$$4^{ag}410=1, \quad Z=\{2, 5, 0^3\};$$

$$3^{mo}510=10, \quad Z=\{4, 3, 0^2, 0\};$$

$$3^{mo}510=4, \quad Z=\{4, 3, 0^3, 0^2\};$$

$$3^{mo}510=2, \quad Z=\{2, 3, 0\};$$

$$3^{mo}510=6, \quad Z=\{2, 3, 0, 0^2\};$$

$$3^{mo}510=6, \quad Z=\{2, 5, 0^3\}.$$

*Asymmetric faces :*

$$4_{as}410=21, \quad 3_{as}510=271.$$

Table D.

*Zoned polar edges :*

$$(33)_{ed.gr}^{2a.d}49=2, \quad Z=\{2.2, 1_p, 0^2, 0_p\},$$

$$Z'=\{2.2, 1_p+2.1, 0_p, 0^{2.1}\};$$

$$(33)_{ed.gr}^{2a.d}49=1, \quad Z=\{2.1, 1_p+2.1, 0_p, 0^{2.1}\},$$

$$Z'=\{2.1, 1_p+2.2, 0_p, 0^{2.1}\};$$

$$(33)_{ed.gr}^{2a.d}49=1, \quad Z=\{2.3, 1_p, 0_p, 0^{2.2}\},$$

$$Z'=\{2.1, 1_p+2.1, 0_p\}.$$

*Zoneless polar edge :*

$$(33)_{ed.gr}^249=1.$$

*Zonal non-polar edges :*

|                   |                         |
|-------------------|-------------------------|
| $(33)_{zo}49=10,$ | $Z=\{4, 3, 0^2, 0\};$   |
| $(33)_{zo}49=6,$  | $Z=\{4, 3, 0^3, 0^2\};$ |
| $(33)_{zo}49=2,$  | $Z=\{2, 3, 0, 0^2\};$   |
| $(33)_{zo}49=8,$  | $Z=\{4, 1, 0^3\};$      |
| $(33)_{zo}49=2,$  | $Z=\{6, 1, 0^5\}.$      |

*Epizonal edges :*

|                                       |                         |
|---------------------------------------|-------------------------|
| $(33)_{ep}49=4,$                      | $Z=\{4, 3, 0^2, 0\};$   |
| $(34)_{ep}39=4,$                      | $Z=\{4, 3, 0^3, 0^2\};$ |
| $(34)_{ep}39=5,$                      | $Z=\{2, 3, 0, 0^2\};$   |
| $(33)_{ep}49=1,$                      | $Z=\{2, 3, 0\};$        |
| $(34)_{ep}39=3, \quad (33)_{ep}49=1,$ | $Z=\{2, 5, 0^3\}.$      |

*Asymmetric edges :*

$$(34)_{as}39=108, \quad (33)_{as}49=352.$$

*Registration of 8-edra 12-acra.*

## Table A.

1. One zoned tetrarchaxine, having for principal poles hexagons and triangles, with amphigrammic secondary axes. The zone is

$$^4Z=\{2.1, 2_p+2_p, 0^{2.1}, 0_{p1}, 0_{p1}\}.$$

2. One 3-zoned homozone monarchaxine, with zoned polar triangles, and zoneless amphigrammic axes. The signatures are read in Table B below.

3. One homozone triaxine, with zoned and zoneless axes all amphigrammic, and signatures next read in Table B.

4. One 6-zoned heterozone monarchaxine, with principal polar hexagons, and secondary axes amphidral and amphigrammic. The zones are first read in Table B below.

5. Three 2-zoned monaxine heteroids, one having polar hexagon and tetragon, with zones

$$Z=\{2.2, 2_p+2.1, 0^{2.1}, 0^{2.1}\},$$

$$Z'=\{2.2, 2_p, 0^{2.1}\},$$

and the others having amphigrammic axes, with the zones

$$Z=\{., 2.2, 0_p^2, 0^{2.1}\}, \quad Z'=\{2.2, 2.2, 0_p^2, 0^{2.1}\}; \text{ and}$$

$$Z=\{2.1, 2.3, 0_p, 0_p, 0^{2.1}\}, \quad Z'=\{2.1, 2.1, 0_p, 0_p\}.$$

6. One zoneless 2-ple monaxine heteroid, with amphigrammic axis.

## 7. Four monozone, of which

2 have the zone  $Z = \{2, 4, 0, 0^3\}$ ,1 has the zone  $Z = \{2, 2, 0, 0\}$ , and1 has the zone  $Z = \{4, 4, 0^2, 0^2\}$ .

## 8. Two asymmetric 8-edra 12-acra.

Table B.

*Heterozone polar faces:*

$$6_{ja}^{6a.d} 6\overline{7} = 1;$$

$$Z = \{\dots, 2_p + 2_p, 0^{2.2}\}, Z' = \{2.2, 2_p, 0_{p''}^2\}, Z'' = \{\dots, 6.1_p, 0_{p''}^6\};$$

$$4_{ja}^{2ag} 8\overline{7} = 1,$$

$$Z = \{2.2, 2_p, 0_p^2\}, Z' = \{\dots, 2_p + 2.2, 0_p^2, 0^{2.2}\}, Z'' = \{\dots, 2_p + 2_{p''}, 0^{2.2}\}.$$

*Homozone polar face:*

$$(8) 3_{obj}^{3mo} 9\overline{7} = 1, \quad Z = \{2.2, 2.1_p + 2.1, 0^{2.1}, 0^{2.1}\}, \\ \zeta = 6\{0_p\}.$$

*Homozone polar edges:*

$$(44)_{ja}^{2a.d} 6\overline{6} = 1, \quad Z = \{2.1, 2.2, 0_p, 0_p, 0^{2.1}\}, \\ \zeta = 4\{0_p\};$$

$$(66)_{ja}^{2a.d} 2\overline{6} = 1, \quad {}^4Z = \{2.1, 2_p + 2_p, 0^{2.1}, 0_{p''}, 0_{p''}\}.$$

*Heterozone polar edge:*

$$(44)_{ja}^{2a.d} 6\overline{6} = 1,$$

$$Z = \{\dots, 2_p + 2.2, 0_p^2\}, Z' = \{2.2, 2_p, 0_p^2\}, Z'' = \{\dots, 2_p + 2_p, 0^{2.2}\}.$$

*Janal zoneless polar edges:*

$$(55)_{ja}^2 4\overline{6} = 1, \quad (55)_{coja}^2 4\overline{6} = 1.$$

Table C.

The polar faces of Table B above are not here repeated.

*Zoned tetrarchipoles:*

$$6_{am.ed.rad}^{3ag} 6\overline{7} = 1, \quad (3) 3_{am.ed}^{3mo} 9\overline{7} = 1, \quad {}^4Z = \{2.1, 2_p + 2_p, 0^{2.1}, 0_p, 0_p\}.$$

These poles are not repeated below.

*Zoned polar faces:*

$$6_{am.ed}^{2a.d} 6\overline{7} = 1, \quad (44)_{am.ed}^{2di} 8\overline{7} = 1, \quad Z = \{2.2, 2.1_p + 2.1, 0^{2.1}, 0^{2.1}\}, \\ Z' = \{2.2, 2_p, 0^{2.1}\}.$$

*Zoned non-polar faces:*

$$\begin{aligned}
7^{mo}57 &= 2, & Z &= \{2, 2, 0, 0\}; \\
6^{ag}67 &= 2, & Z &= \{2, 4, 0, 0^3\}; \\
6^{ag}67 &= 1, & Z &= \{\dots, 4, 0^4\}; \\
5^{mo}77 &= 3, & Z &= \{4, 4, 0^2, 0^2\}; \\
5^{mo}77 &= 3, & Z &= \{2, 4, 0, 0^3\}; \\
5^{mo}77 &= 1, & Z &= \{2, 2, 0, 0\}; \\
4^{di}87 &= 1, & Z &= \{4, 4, 0^2, 0^2\}; \\
4^{ag}87 &= 3, & Z &= \{2, 4, 0, 0^3\}; \\
4^{ag}87 &= 1, & Z &= \{\dots, 4, 0^4\}; \\
4^{ag}87 &= 1, & Z &= \{4, 4, 0^2, 0^2\}; \\
4^{ag}47 &= 2, & Z &= \{2, 6, 0, 0^5\}; \\
3^{mo}97 &= 2, & Z &= \{2, 4, 0, 0^3\}; \\
3^{mo}97 &= 1, & Z &= \{2, 6, 0, 0^5\}; \\
3^{mo}97 &= 3, & Z &= \{4, 4, 0^2, 0^2\}.
\end{aligned}$$

*Objanal monozone face:*

$$5_{obj}^{mo}77 = 1, \quad Z = \{2.2, 2.2, 0^{2.1}, 0^{2.1}\},$$

which is also above enumerated.

*Asymmetric faces:*

$$\begin{aligned}
7_{as}57 &= 2, & 6_{as}67 &= 5; \\
5_{as}77 &= 7, & 4_{as}87 &= 8, & 3_{as}97 &= 8.
\end{aligned}$$

## Table D,

not containing the edges of the preceding Table B.

*Zoned polar edges:*

$$\begin{aligned}
(66)_{am.gr}^{2a.d}26 &= 1, & (44)_{am.gr}^{2a.d}66 &= 1, & Z &= \{2.2, 2.2, 0_p^2, 0^{2.1}\}, \\
& & & & Z' &= \{\dots, 2.2, 0_p^2, 0^{2.1}\}; \\
(77)_{am.gr}^{2a.d}06 &= 1, & (44)_{am.gr}^{2a.d} &= 1, & Z &= \{2.1, 2.3, 0_p, 0_p, 0^{2.1}\}, \\
& & & & Z' &= \{2.1, 2.1, 0_p, 0_p\}.
\end{aligned}$$

*Zoneless polar edges:*

$$(66)_{am.gr}^226 = 1, \quad (55)_{am.gr}^246 = 1.$$

*Zonal non-polar edges:*

$$\begin{aligned}
(44)_{zo}66 &= 1, & (66)_{zo}26 &= 1, & (55)_{zo}46 &= 1, & Z &= \{2, 4, 0, 0^3\}; \\
(55)_{zo}46 &= 1, & & & & & Z &= \{2, 2, 0, 0\}; \\
(66)_{zo}26 &= 1, & (55)_{zo}46 &= 2, & & & Z &= \{4, 4, 0^2, 0^2\}; \\
(55)_{zo}46 &= 1, & & & & & Z &= \{4, 2, 0^2\}.
\end{aligned}$$

*Objanal zonal edge:*

$$(55)_{zob}46=1,$$

$$Z=\{2.2, 2.2, 0^{2.1}, 0^{2.1}\}.$$

This edge is one of the zonals above entered.

*Epizonal edges:*

$$\begin{aligned} (63)_{ep}56=2, \quad (54)_{ep}56=2, \quad (65)_{ep}36=1, \\ (64)_{ep}46=2, \quad (43)_{ep}76=1, \end{aligned} \left. \vphantom{\begin{aligned} (63)_{ep}56=2, \quad (54)_{ep}56=2, \quad (65)_{ep}36=1, \\ (64)_{ep}46=2, \quad (43)_{ep}76=1, \end{aligned}} \right\} Z=\{2, 4, 0, 0^3\};$$

$$(57)_{ep}26=1, \quad Z=\{2, 2, 0, 0\};$$

$$(63)_{ep}56=1, \quad (53)_{ep}66=2, \quad (54)_{ep}56=1, \quad (34)_{ep}76=1, \quad Z=\{4, 4, 0^2, 0^2\};$$

$$(64)_{ep}46=2, \quad Z=\{., 4, 0^4\};$$

$$(44)_{ep}66=1, \quad (34)_{ep}76=1, \quad Z=\{2, 6, 0, 0^5\}.$$

*Asymmetric edges:*

$$(73)_{as}46=7, \quad (74)_{as}36=7, \quad (75)_{as}26=4, \quad (76)_{as}16=2;$$

$$(66)_{as}26=1, \quad (65)_{as}36=10, \quad (64)_{as}46=11, \quad (63)_{as}56=9;$$

$$(55)_{as}46=3, \quad (54)_{as}56=12, \quad (53)_{as}66=9;$$

$$(44)_{as}66=3, \quad (43)_{as}76=5.$$

### *Registration of 12-edra 8-acra.*

Table A.

1. One zoned tetrarchaxine, having for principal poles hexaces and triaces, with amphigrammic secondary axes. The zone is

$$^4Z=\{2_p+2_p, 2.1, 0_p, 0_p, 0^{2.1}\}.$$

2. One 3-zoned homozone monarchaxine, with zoned polar triaces and zoneless amphigrammic axes. The zonal and zonoid signatures are

$$Z=\{2.1_p+2.1, 2.2, 0^{2.1}, 0^{2.1}\}, \quad \zeta=6\{0_p\}.$$

3. One homozone triaxine with zoned and zoneless amphigrammic axes, and zonal signature first below read in Table B.

4. One 6-zoned monarchaxine heterozone, with principal polar hexaces, and secondary axes amphigonal and amphigrammic. The zonal signatures are

$$Z=\{2_p+2_p, 0^{2.2}\}, \quad Z'=\{2_p, 2.2, 0_p^2\}, \quad Z''=6\{1_p, \dots, 0_{p_n}\}.$$

5. Three 2-zoned monaxine heteroids, one having polar hexace and tessarace, with the zones

$$Z=\{2_p+2.1, 2.2, 0^{2.1}, 0^{2.1}\}, \quad Z'=\{2_p, 2.2, 0^{2.1}\},$$

and two having amphigrammic axes, with the signatures

$$Z=\{2.2, \dots, 0_p^2, 0^{2.1}\}, \quad Z'=\{2.2, 2.2, 0_p^2, 0^{2.1}\}, \text{ and}$$

$$Z=\{2.3, 2.1, 0_p, 0_p, 0^{2.1}\}, \quad Z'=\{2.1, 2.1, 0_p, 0_p\}.$$

6. One zoneless 2-ple monaxine heteroid, with amphigrammic axis.

7. Four monozones, of which

2 have the zone  $Z = \{4, 2, 0, 0^3\}$ ,

1 has the zone  $Z = \{2, 2, 0, 0\}$ ,

1 has the zone  $Z = \{4, 4, 0^2, 0^2\}$ .

8. Two asymmetric 12-edra 8-acra.

Table B.

*Homozone polar edges :*

$$(33)_{ja}^{2a.d} 410 = 2, \quad Z = \{2.2, 2.1, 0_p, 0_p, 0^{2.1}\}.$$

*Heterozone janal polar edge :*

$$(33)_{ja}^{2a.d} 410 = 1,$$

$$Z = \{2_p + 2.2, \dots, 0_p^2\}, \quad Z' = \{2_p, 2.2, 0_p^2\}, \quad Z'' = \{2_p + 2_p, \dots, 0^{2.2}\}.$$

*Janal zoneless polar edges :*

$$(33)_{ja}^2 410 = 1, \quad (33)_{coja}^2 410 = 1.$$

Table C.

*Zoned non-polar faces :*

$$3^{mo} 511 = 10, \quad Z = \{4, 4, 0^2, 0^2\};$$

$$3^{mo} 511 = 3, \quad Z = \{2, 4, 0^2\};$$

$$3^{mo} 511 = 5, \quad Z = \{4, 2, 0^2, 0^2\};$$

$$3^{mo} 511 = 1, \quad Z = \{4, 2, 0^3, 0\};$$

$$3^{mo} 511 = 3, \quad Z = \{2, 2, 0, 0\};$$

$$3^{mo} 511 = 1, \quad Z = \{6, 2, 0, 0^5\}.$$

*Objanal monozone faces :*

$$3^{mo} 511 = 2, \quad Z = \{2.2, 2.2, 0^{2.1}, 0^{2.1}\},$$

which are also above entered.

*Asymmetric face :*

$$3_{as} 511 = 55.$$

Table D,

not comprising the edges in the above Table B.

*Zoned polar edges :*

$$(33)_{am.gr}^{2a.d} 410 = 2, \quad Z = \{2.2, 2.2, 0_p^2, 0^{2.1}\},$$

$$Z' = \{2.2, \dots, 0_p^2, 0^{2.1}\};$$

$$(33)_{am.gr}^{2a.d} 410 = 2, \quad Z = \{2.3, 2.1, 0_p, 0_p, 0^{2.1}\},$$

$$Z' = \{2.1, 2.1, 0_p, 0_p\}.$$

*Zoneless polar edge :*

$$(33)_{am,gr}^2 410 = 2.$$

*Zonal non-polar edges :*

$$(33)_{zo} 410 = 8, \quad Z = \{4, 2, 0, 0^3\};$$

$$(33)_{zo} 410 = 1, \quad Z = \{2, 2, 0, 0\};$$

$$(33)_{zo} 410 = 5, \quad Z = \{4, 4, 0^2, 0^2\};$$

$$(33)_{zo} 410 = 2, \quad Z = \{4, \dots, 0^4\};$$

$$(33)_{zo} 410 = 2, \quad Z = \{6, 2, 0, 0^5\}.$$

*Objanal zonal edge :*

$$(33)_{zo.ob} 410 = 1, \quad Z = \{2.2, 2.2, 0^{2.1}, 0^{2.1}\},$$

which is also above entered as zonal.

*Epizonal edges :*

$$(33)_{ep} 410 = 3, \quad Z = \{4, 2, 0, 0^3\};$$

$$(33)_{ep} 410 = 1, \quad Z = \{2, 2, 0, 0\};$$

$$(33)_{ep} 410 = 3, \quad Z = \{4, 4, 0^2, 0^2\};$$

$$(33)_{ep} 410 = 1, \quad Z = \{2, 4, 0^2\}.$$

*Asymmetric edge :*

$$(33)_{as} 410 = 83.$$

I may be permitted to remark here that these results were in my possession early in 1858, when the prize question of the French Academy was published for the competition of 1861 : "*Perfectionner en quelque point important la théorie géométrique des polyèdres.*" My work on this theory was first completely composed in the French language, in its present form, with the intention of presenting it to the Academy in 1861. Any person, who cares to know the reasons why I altered its destination, may read them at page 352 of the 'Memoirs of the Literary and Philosophical Society of Manchester,' 3rd series, vol. i. 1862, beginning at the second line *ab infra*.

## II. "Contributions towards the History of the Monamines.—

No. VI. Note on the Action of Iodide of Methyl on Ammonia." By A. W. HOFMANN, LL.D., F.R.S. Received December 2, 1862.

When studying, many years ago, the action of iodide of methyl upon ammonia, I pointed out the existence of dimethylamine among the products of the reaction. The amount of iodide of dimethylam-